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Intertemporal Modeling: Computable General Equilibrium and Environmental Applications

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Intertemporal Modeling: Computable General Equilibrium and Environmental Applications

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The first chapter uses an intertemporal optimal control model to analyze the problem of transboundary pollution that has both stock and flow characteristics. In this theoretical model, firms produce a flow pollutant that adversely affects the state in which it was released and downstream states. The pollution can be cleaned-up by removing the damaging elements from the flow before they enter the environment, but the removed harmful elements do not simply disappear. Their disposal generates a stock pollutant that only affects the state in which it was generated. The model is solved for the optimal time path for cleanup of the flow pollutant, and the Pigouvian tax rate that will achieve the optimal result. In a transboundary setting, regulation chosen by the upstream polluting state will clean up less than the optimal amount of flow pollutant. The

model also demonstrates that a federal regulation that ignores the stock pollutant will clean up more than the optimal amount of the flow pollutant.

The second chapter presents an econometrically-estimated intertemporal computable general equilibrium model with labor and capital adjustment costs. Computable general equilibrium models are widely used for evaluating policies, but they generally fail to capture short run rigidities, especially in labor markets. This shortcoming has led to the continued use of old style reduced-form macro models for policy analysis in situations where short-run rigidities are likely to be important. Capturing labor market rigidities is particularly important in the analysis of environmental regulations or other policies that strongly affect narrow sections of the economy. In the long run, the industries adversely affected by such policies will shrink. In the short run, however, the labor employed in those industries is not able to move. The true cost of such a policy, therefore, depends critically on the transition path of the economy from the announcement of the policy to the new long run equilibrium. Including labor adjustment costs addresses this problem and will allow computable general equilibrium models to be used for a much broader range of policy analyses than they have been in the past.

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Chapter 1: Transboundary Pollution, When Flow Reduction Generates Stock Externalities

1-1 INTRODUCTION

Traditional models of pollution abatement typically examine either a stock pollutant or a flow pollutant. While this may be appropriate for many scenarios, simply classifying a pollutant as a stock or a flow is inappropriate for a whole class of environmental problems. Whenever the reduction of a flow pollutant is achieved by removing the pollutant from emissions before it enters the environment, instead of reducing the amount of the polluting input used, the actual pollutant does not disappear. The removed pollutant still needs to be disposed of, and it may collect as a stock. Any further damages caused by this stock of pollution need to be accounted for by any policy designed to control the flow pollutant.

Some have researched polluting activities that generate both stock and flow externalities. Wirl (1994) looks at Pigouvian taxation of energy production that generates both stock and flow externalities, where the market is characterized by strategic noncompetitive energy pricing. In this case, energy generation produces carbon dioxide, a stock pollutant that contributes to global warming, and a flow pollutant in the form of acid rain. Sandal and Steinshamn (1998) consider an optimal corrective tax as a closed form feedback control law when production of a consumer good generates both flow and stock externalities. Aronsson (1998) investigates green tax reform in a dynamic general equilibrium model, and relates the welfare effects of changes in the tax mix to responses in emission flow and the resulting stock of pollution, as well as employment and capital stock responses along the general equilibrium path. In this case, the stock of pollution causes the damages and the flow of emissions simply generates the stock. Keohane *et al.* (2000) define environmental quality as a stock variable. They then consider policies with two instruments to improve environmental quality, curbing the flow of deterioration and restoring the stock of quality, and find that the optimal policy employs both instruments.

All of the above models allow the polluting activity to generate both flow and stock pollutants, or they let the flow pollutant simply generate the stock pollutant. This chapter considers the case where it is the efforts to *reduce* the flow externality that actually generate the stock externality. This situation could occur when abatement of flow pollution is accomplished by cleaning the flow or removing the harmful elements before the flow is released. In this case, the pollution removed from the flow still exists, and it accumulates as a stock of pollution. If this new stock is potentially harmful, and dealing with it is nontrivial, then the generation of this stock pollutant should be considered when designing regulations to control the flow pollutant. Several different kinds of pollution problems fit this description, including air pollution, such as sulfur dioxide, reduced by use of a scrubber that concentrates the sulfur into a highly-toxic solid waste that must be stored. Another motivating example for this chapter is non-point source nitrogen run-off from agricultural land. This run-off collects in streams and rivers where it is a flow pollutant. One method of controlling nitrogen run-off is to plant riparian buffers where plants can absorb the excess nitrogen before it reaches waterways, but another method is to introduce no-till farming, which reduces erosion and nitrogen run-off. While no-till farming reduces nitrogen run-off, it has also been linked to increased levels of nitrate leaching into the ground water (Taylor *et al.* 1992). In this situation the flow pollutant is prevented from entering the waterway, but some of it is converted to a stock pollutant when it collects in the ground water.

In order to address these issues, this chapter uses a dynamic optimal control with a single control variable, representing the percentage of pollution removed from emissions, and a single state variable, which describes the stock pollution resulting from the cleaned and diverted flow. The second section of this chapter builds the model and describes some of its behavioral characteristics. In the third section of this chapter, we find the optimal Pigouvian tax on emissions and examine what happens when policy makers cannot observe, or simply ignore, the stock pollutant.

Another contribution of this chapter, presented in the fourth section, is to consider the transformation of a flow pollutant into a stock pollutant in a transboundary context.[1] In this case, the flow pollutant harms the state where it is generated as well as downstream states that it flows into. If the flow pollutant is cleaned up and some of it is transformed into a stock pollutant, that stock pollutant stays put and only affects the state where it is created. We revisit the

[1] Other papers have looked at transboundary pollution in federal versus decentralized settings. Silva and Caplan (1997) discuss transboundary pollution control in a federal system from a game theoretic standpoint. Silva (1997) looks at decentralized control of transboundary pollution when pollution abatement is undertaken for the sole intent of deterring immigration.

regulatory issues in this section from both a state and federal perspective. In this situation we would expect a state regulator to under-control the flow pollutant in order to avoid the stock pollutant and thus to export the pollution to downstream states. Federal regulators are able to implement the optimal policy if they have perfect information about both types of pollutant. We also consider the case where the federal regulator ignores the stock pollutant. In this situation federal regulators will over-control the flow pollutant. In this setting, an optimal outcome can only be achieved if Coase payments between upstream and downstream states are possible.

The fifth section presents the chapter's conclusion and suggestions for further research. A final twist in this chapter is to flip the roles of the flow and stock pollutants so that reducing the stock of pollution generates a flow pollutant. One example of this is a landfill where reducing the stock of garbage could involve incinerating it, which in turn generates a flow pollutant. This analysis is presented in Appendix 1.5 along with implications for the transboundary case.

1-2 THE MODEL

The model in this chapter is an optimal control model in continuous time with one state and one control variable. This model contains N firms, each of which emits a constant flow of emissions $\bar{x}$. These emissions are not a choice variable in this model. This restriction simplifies the math by limiting the model to one control variable. The fixed emissions case examined here could reasonably apply to industries that inelasticly demand the potentially polluting input. The one control variable in this model is r, the fraction of pollution removed from the emissions. The total flow-pollutant is thus $N(1\text{-}r)\bar{x}$. The key to this model is that the pollution removed from the flow of emissions does not simply disappear. Instead, a fraction (θ) of the removed pollutant remains in a stock of pollution (s), while the remainder is rendered harmless.

With the above notation, the equation of motion for the stock-pollutant can now be written as:

1.1 $\dot{s} = \theta N r \bar{x} - \delta s$,

where δ is the decay rate of the stock. The damage from the flow-pollutant is a function of the total amount of flow-pollutant, $g[N(1-r)\bar{x}]$, where g' > 0 and g''>0 both hold. The damage from the stock-pollutant as a function of the stock is h(s), where h'(s)>0 and h''(s)>0. The cost per firm of cleaning up emissions is a function of the level of emissions from each firm and the percentage of pollution removed from the emissions, $c(r, \bar{x})$ where $\frac{\partial c}{\partial r} > 0$ and $\frac{\partial^2 c}{\partial r^2} > 0$. Finally, $\pi(\bar{x})$ is the profit level before cleanup as a function of emissions. In this version of the model emissions are held constant, so the profits are constant as well.

The social planner's problem can now be written as:

1.2 $\max_{r} \int_0^{\infty} \left[N\pi(\bar{x}) - g\left(N(1-r)\bar{x}\right) - h(s) - Nc\left(r, \bar{x}\right)\right] e^{-\rho t} dt \quad \text{s.t.} \quad \dot{s} = \theta N r \bar{x} - \delta s$, where t is time and ρ is the interest rate. The Hamiltonian is:

1.3 $$H = \left[N\pi(\bar{x}) - g(N(1-r)\bar{x}) - h(s) - Nc(r,\bar{x})\right]e^{\rho t} + \Lambda\left(N\theta r\bar{x} - \delta s\right).$$

Performing the maximization results in the following first order conditions:

1.4 $$\frac{\partial H}{\partial r} = N\bar{x}g'\left(N(1-r)\bar{x}\right)e^{-\rho t} - N\frac{\partial c}{\partial r}e^{-\rho t} + \Lambda N\theta\bar{x} = 0$$

1.5 $$\frac{\partial H}{\partial s} = -h'(s)e^{-\rho t} - \Lambda\delta = -\dot{\Lambda}$$

1.6 $$\frac{\partial H}{\partial \Lambda} = N\theta r\bar{x} - \delta s = \dot{s}\,.$$

Since this is an infinite horizon problem, a convenient transformation is to define λ such that $\Lambda = \lambda e^{-\rho t}$, where Λ is the present value multiplier and λ is the current value multiplier. The value of the damages in year t from a small change in the stock-pollutant in year t is λ, the current value of marginal damages from the stock-pollutant. Using this transformation, the first order conditions can now be written as:

1.7 $$\bar{x}g'\left(N(1-r)\bar{x}\right) - \frac{\partial c}{\partial r} + \lambda\theta\bar{x} = 0$$

1.8 $$-h'(s) - \lambda\delta = -\dot{\lambda} + \rho\lambda$$

1.9 $$\dot{s} = N\theta r\bar{x} - \delta s\,.$$

The equation of motion for the stock variable (s) is shown in 1.9, and the equation of motion for the control variable (r) can be found using 1.7 and 1.8. To do this, we must first take the time derivative of 1.7, which results in:

$$1.10 \qquad \dot{\lambda} = \frac{N\bar{x}^2 g''\left(N(1-r)\bar{x}\right)\dot{r} + \frac{\partial^2 c}{\partial r^2}\dot{r}}{\theta\bar{x}} .$$

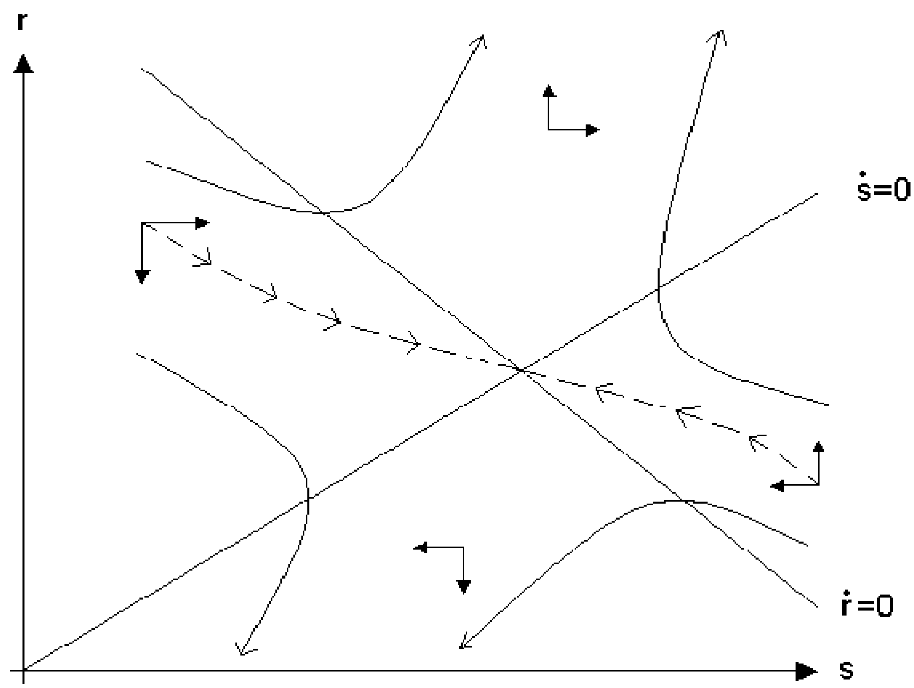
Then we can substitute the expression for $\dot{\lambda}$ from 1.10 into 1.8 and solve for $\dot{r}$. The resulting equation of motion for the control variable r is:

$$1.11 \qquad \dot{r} = \frac{(\rho+\delta)\left[-\bar{x}g'\left(N(1-r)\bar{x}\right) + \frac{\partial c}{\partial r}\right] + \theta\bar{x}h'(s)}{N\bar{x}^2 g''\left(N(1-r)\bar{x}\right) + \frac{\partial^2 c}{\partial r^2}} .$$

It can be shown that the above equations of motion describe a saddle path stable system of differential equations.[2] Given this saddle path stability, the assumption that the transversality condition holds, imposes that the model approaches the steady state as t approaches infinity. The system can best be understood by use of the phase diagram in Figure 1.1.[3]

[2] Confirmation of saddle path stability is shown in Appendix 1.1.
[3] The signs of the slopes of the isoclines are derived in Appendix 1.2.

Figure 1.1 Phase Diagram for Removal (r) and Stock (s)



The saddle path in the phase diagram depicts how the fraction of pollution removed from emissions, r, and the stock of pollution, s, optimally evolve over time. If initially the level of the stock-pollutant is too low, then the fraction of pollution removed from emissions will optimally begin at a high level. Following the optimal path, the stock-pollutant will grow since the amount contributed to it from the cleanup activity is greater than the decay rate. Optimally, the percentage of pollution removed from emissions should be reduced over time, and the growth of the stock-pollutant slows until the steady state equilibrium is reached. If the initial stock is too large, then the process is reversed. The fraction of pollution removed from emissions optimally begins at a low level to allow the stock to decay. As the stock decays, more pollution should be removed from emissions and the steady state equilibrium is approached.

We now perform an experiment to see how the steady state levels of s and r change when changing θ (the fraction of the removed pollutant that adds to the stock). This can be interpreted as an improvement in the cleanup technology that reduces the percentage of removed emissions that contribute to the stock-pollutant. To perform this analysis, we take the equations of motion for r and s, expressed as functions of r, s, and θ, and set them equal to zero:

1.12 $$\dot{r} \equiv f(r,s,\theta) = \frac{(\rho+\delta)\left[-\bar{x}g'\left(N(1-r)\bar{x}\right)+\dfrac{\partial c}{\partial r}\right]+\theta\bar{x}h'(s)}{N\bar{x}^2 g''\left(N(1-r)\bar{x}\right)+\dfrac{\partial^2 c}{\partial r^2}} = 0$$

1.13 $$\dot{s} \equiv k(r,s,\theta) = N\theta r\bar{x} - \delta s = 0\,.$$

Totally differentiate these equations and express partial derivatives as subscripts to get:

1.14 $$f_r dr + f_s ds + f_\theta d\theta = 0$$

1.15 $$k_r dr + k_s ds + k_\theta d\theta = 0\,.$$

Rearrange 1.15 to isolate dr and substitute the resulting expression into 1.14 to eliminate dr to get:

1.16 $$-\frac{k_s f_r}{k_r} ds - \frac{k_\theta f_r}{k_r} d\theta + f_s ds + f_\theta d\theta = 0 .$$

Collect terms and rearrange to solve for ds/dθ results in:[4]

1.17 $$\frac{ds}{d\theta} = \frac{\frac{k_\theta f_r}{k_r} - f_\theta}{-\frac{k_s f_r}{k_r} + f_s} \gtrless 0.$$

This result shows that reducing θ has an indeterminate effect on the stock. All else held equal, the direct effect of reducing θ is to lower the stock-pollutant, since less of the cleaned emissions end up in the stock. However, it is possible that the optimal response to the better cleanup technology is to increase the percentage pollution removed from the flow so much that the amount of stock-pollutant actually increases.

Now attention is turned to the effect of a decrease in θ on the level of r at the steady state. Rearrange 1.15 to isolate ds and substitute the resulting expression into 1.14 to eliminate ds to get:

[4] The signs of ds/dθ and dr/dθ are found in Appendix 1.3.

1.18 $$f_r dr - \frac{f_s k_r}{k_s} dr - \frac{f_s k_\theta}{k_s} d\theta + f_\theta d\theta = 0\,.$$

Now collect terms and rearrange to solve for dr/dθ:

1.19 $$\frac{dr}{d\theta} = \frac{-f_\theta + \dfrac{f_s k_\theta}{k_s}}{f_r - \dfrac{f_s k_r}{k_s}} < 0\,.$$

A reduction of θ definitively increases r, the fraction of pollution removed from emissions. The improved cleanup technology, lower θ, implies that less of the removed flow pollutant, $Nr\bar{x}$, ends up in the stock of pollution. Since the negative externality associated with removing a fraction of the flow-pollutant has decreased, the optimal fraction of pollution removed from emissions, r, has increased.

The effects of the change in θ can be seen graphically in the phase diagram shown in Figure 1.2.[5]

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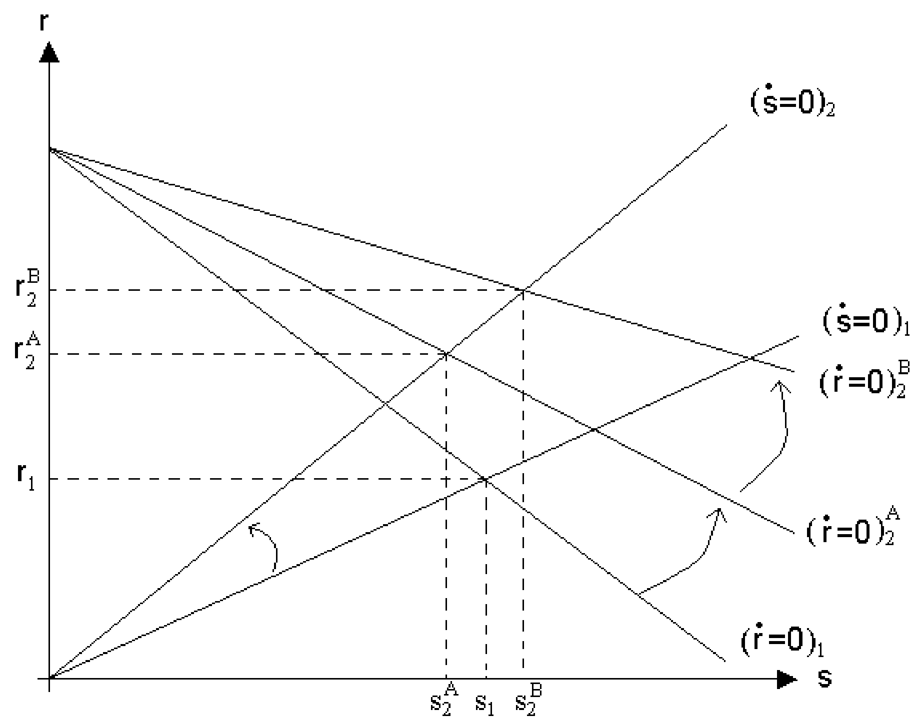
[5] The changes in the slope of the isoclines are derived in Appendix 1.3.

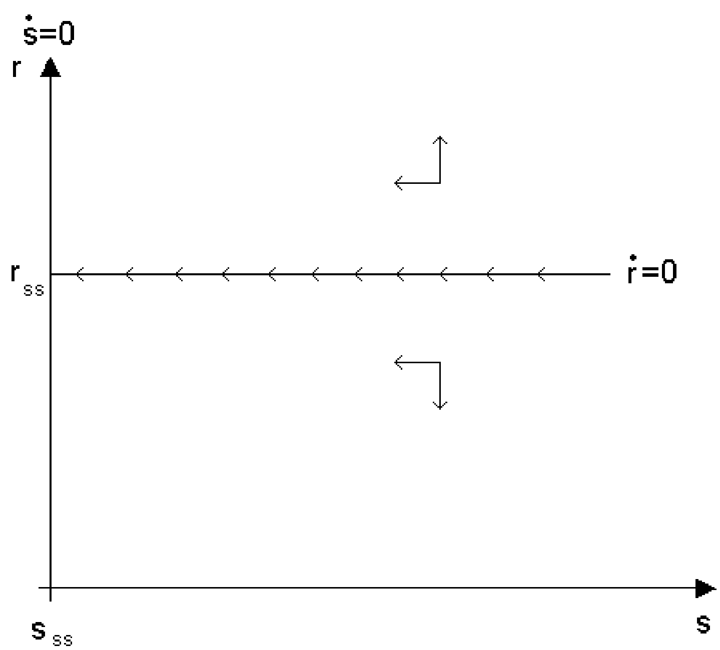
Figure 1.2 The Effect of Changes in θ on the s and r Isoclines.



This shows how the steady state levels of r and s respond to a decrease in θ. A decrease in s is associated with a small increase in r and an increase in s is associated with a larger increase in r. The initial response to the change in θ is an immediate increase in the level of r to reach the new saddle path. The system will then evolve along the new saddle path and approach the new steady state equilibrium. Figure 1.2 accurately depicts the $\dot{s} = 0$ isocline as linear and passing through the origin. However, the $\dot{r} = 0$ isocline is not necessarily linear and does not necessarily rotate about a single vertical intercept as depicted in Figure 1.2.

Only the change in slope of the $\dot{r} = 0$ isocline is certain without assuming particular functional forms. These limitations imply that no simple condition determines if $ds/d\theta$ is positive or negative.[6] We can gain a little more understanding of the model by looking at the extreme case where $\theta = 0$. This case is depicted in Figure 1.3.

Figure 1.3 Phase Diagram for Removal (r) and Stock (s) with θ = 0.



The $\dot{s} = 0$ isocline becomes vertical, while the slope of the $\dot{r} = 0$ isocline falls to zero. In this case, cleaning up the flow pollutant now contributes nothing to the

[6] The expression that governs the sign of $ds/d\theta$ is found in Appendix 1.3.

stock. Therefore the r will immediately jump to the new steady state level and the stock approaches zero according to the rate of decay.

1-3 POLICIES

Now we turn our attention to what policy will allow society to reach the optimum and the effects of non-optimal policies. In a free market with no tax, the firm will set r=0 and not clean emissions at all since $\frac{\partial c}{\partial r} > 0$. However, a properly designed policy will cause the firm to choose the optimal path for r. The transformed first order condition in 1.7 gives the condition that must be satisfied by the optimal policy. This condition is rearranged to give us 1.20:

$$\bar{x} g'\left(N(1-r)\bar{x}\right) + \lambda\theta\bar{x} = \frac{\partial c}{\partial r}. \tag{1.20}$$

The first term on the left hand side of 1.20 is the marginal damage from the flow-pollutant for a small change in r, the fraction of pollution removed from emissions. The value of this term is positive. The second term is the present value of the marginal damages from the stock-pollutant for a small change in r. This term is negative since $0 \le \theta \le 1$, $\bar{x} \ge 0$ and $\lambda<0$. The term on the right hand side of this expression is the marginal cost of increasing the fraction of pollution removed from emissions, which is a positive number. If the left hand side of 1.20 is greater than zero when evaluated at r=0, then an optimal level of cleanup exists $0 < r \le 1$. If the left hand side of 1.20 is less than or equal to zero, then this condition will not hold since $\frac{\partial c}{\partial r} > 0$. If this is the case, then the optimal policy is no cleanup (i.e. set r=0). If increasing the stock-pollutant causes greater harm than the benefit from reducing the flow pollutant, then it is better to do nothing.

A tax on cleaned emissions, τ, can be set to satisfy the condition in 1.20. The firm's problem is to choose r to maximize profits after tax and cleanup cost:

$$1.21 \qquad \max_{r} f(\bar{x}) - c(r, \bar{x}) - (1 - r)\bar{x}\tau .$$

From this optimization, the firm will choose r to satisfy the condition:

$$1.22 \qquad \frac{\partial c}{\partial r} = \bar{x}\tau .$$

So, using the above equation and the optimality condition in 1.20 the resulting optimal tax is:

$$1.23 \qquad \tau^{*} = \theta\lambda + g'\left(N(1 - r)\bar{x}\right) .$$

This optimal tax rate varies over time, as λ and other variables vary with time. The first term on the right hand side is the present value of marginal damages from an increase in the stock-pollutant at any moment in time. This term is negative since λ is always less than zero, and θ is bounded by zero and one. The second term on the right hand side is the marginal damage from an increase in the flow pollutant, which is a positive number. The optimal tax will vary over time because λ changes with time.[7] The firm's response to the optimal tax is to choose r to follow the saddle path shown in Figure 1.1.

Now consider what happens when the regulator ignores the damages from the stock-pollutant. In this situation the regulator is only concerned about the flow pollutant, so the regulation is designed to satisfy the condition:

$$1.24 \qquad \bar{x}g'\left(N(1-r)\bar{x}\right) = \frac{\partial c}{\partial r}.$$

This is equivalent to equation 1.20, with the exception that λ now is equal to zero. The tax that will satisfy this condition is:

$$1.25 \qquad \tau^f = g'\left(N(1-r)\bar{x}\right).$$

This tax on just the flow pollutant omits the $\theta\lambda$ term from the optimal tax, which accounted for the stock-pollutant and generated the variability in the tax rate. Since λ is less than zero, this tax will be greater than the optimal tax. The resulting level of cleanup r^f will be strictly greater than the optimal rate of cleanup r^*. Figure 1.4 below shows a phase diagram with both the optimal path and the path resulting from the tax ignoring the stock-pollutant.

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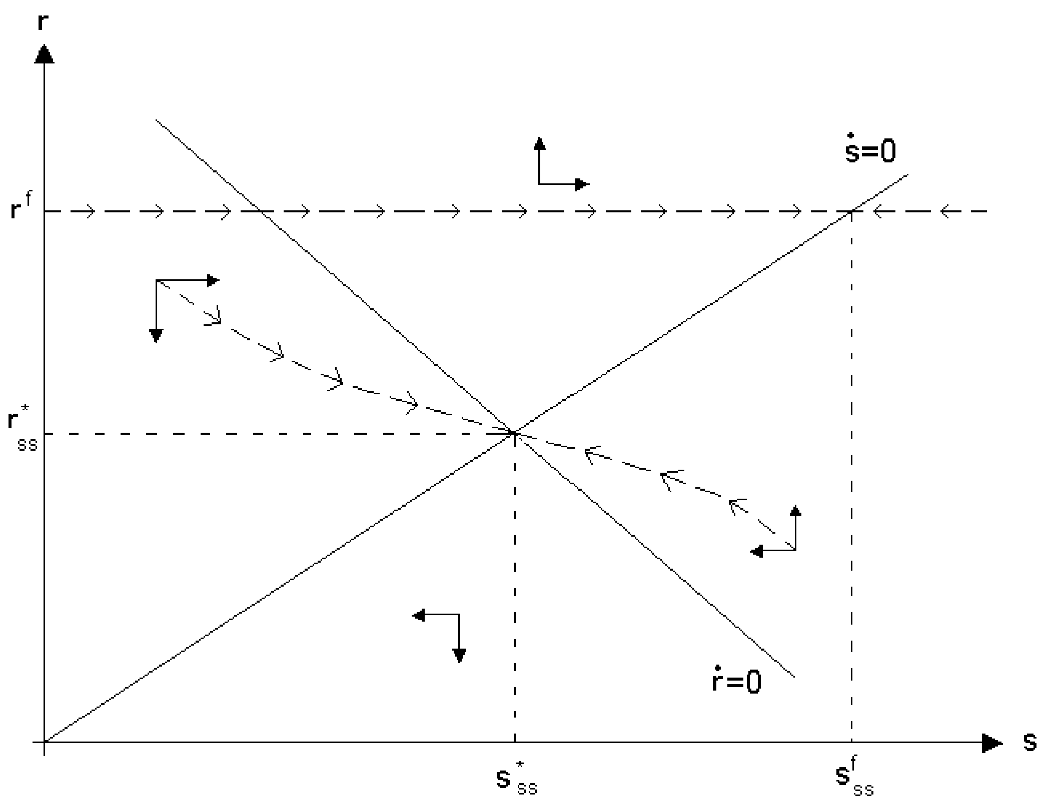
[7] The equation of motion for λ is derived in Appendix 1.4.

Figure 1.4 Phase Diagram When Regulator Ignores Stock-Pollutant



When the regulator ignores the stock-pollutant, the level of cleanup is a constant that is higher than anywhere on the optimal path. Since r is held constant, the path over time in the phase diagram is simply a horizontal line with the stock-pollutant evolving to a steady state higher than the optimal steady state level.

1-4 TRANSBOUNDARY FLOW

Now we divide the affected region into an upstream and a downstream state, in order to introduce transboundary problems into the model. The flow pollutant is emitted in the upstream state and travels to the downstream state.

However, the stock-pollutant remains entirely in the upstream state. This modification of the model is achieved by adding the variable γ, which is the percentage of flow pollutant damage affecting the upstream state. The total amount of flow pollution damage affecting the upstream state is thus $\gamma g\left(N(1-r)\bar{x}\right)$, and the amount of damage affecting the downstream state is $(1-\gamma)g\left(N(1-r)\bar{x}\right)$. With this distinction, we can look at how a tax from a self-interested regulator in the upstream state compares to the optimal tax. The tax set by the upstream state will be:

1.26 $$\tau^{up} = \theta\lambda + \gamma g'\left(N(1-r)\bar{x}\right).$$

This is identical to the optimal tax except for the addition of γ, which limits the second term of the right hand side to the marginal damage to the upstream state from an increase in the flow pollutant. Since $0 \le \gamma \le 1$ and g'()>0, the tax implemented by the upstream state is less than or equal to the optimal tax. This implies that the amount of cleanup, r, will be below the optimal level.

Now consider the case of a federal government that ignores the stock-pollutant, or is in an information environment where it believes the there are no damages from the stock-pollutant. If the information advantage of the state government over the federal government is the ability to observe the stock-pollutant, then the federal government is in the same position as the naïve regulator in the previous section. Since the federal government ignores the stock-pollutant, the federal tax will be the same as the tax on just the flow-pollutant in equation 1.25:

$$1.27 \qquad t^f = g'\left(N(1-r)\bar{x}\right) .$$

This tax accounts for the damages from the flow pollutant to the upstream and downstream state, but misses the damages from the stock-pollutant. As discussed before, this tax is too large and results in too much cleanup. The following inequalities sum up the dilemma faced in this situation:

$$1.28 \qquad \begin{matrix} \tau^{up} \le \tau^* \le \tau^f \\ r^{up} \le r^* \le r^f \end{matrix} .$$

The local regulator has the best information, but ignores the downstream effects. This results in a tax that is too low and too little cleanup. The upstream state is quite happy with this situation, because it is able to export more of its pollution to the downstream state than at the optimum. The federal government can account for the downstream pollution; however, since it is unaware of the stock-pollutant, it is unable to account for it. This results in a tax that is too high and too much cleanup. This situation benefits the downstream state at the expense of the upstream state. The downstream state is made better off by the excess reduction of the flow pollutant, while the upstream state suffers from the increased amounts of the stock-pollutant.

In order to reach an optimum in the setting where only the state government is aware of the stock-pollutant, a mechanism must exist to allow for Coase payments between the states.[8] If property rights are granted to the upstream state, then regulators from the upstream state would initially set a Pigouvian tax rate equal to τ^{up} (1.26). If the downstream state is allowed to make Coase payments that subsidize the firm's cleanup efforts, then the firm's new maximization problem is:

1.29 $$\max_{r} f\left(\bar{x}\right) - c\left(r, \bar{x}\right) - \left(1 - r\right) \cdot \bar{x} \cdot \tau^{up} + r \cdot \bar{x} \cdot sub^{dwn},$$

where sub^{dwn} is the price of the downstream state's cleanup subsidy. The downstream state will be willing to pay the following subsidy to each firm for every unit of flow pollutant it removes ($r\bar{x}$):

1.30 $$sub^{dwn} = (1 - \gamma) g'\left(N\left(1 - r\right)\bar{x}\right).$$

The combination of this subsidy and the upstream state's Pigouvian tax will optimally control the flow pollutant.

[8] There are a number of ways to set up the Coase payments. While the presentation here grants property rights to the upstream state and has the downstream state pay the firm directly, it would also be possible to have payments directly to the state, and to assign property rights to either state.

1-5 CONCLUSION

This chapter has shown that in situations where cleaning up a flow pollutant produces a stock-pollutant as a byproduct, consideration of that stock-pollutant is important for designing an optimal policy to control the pollutant. Furthermore, this chapter has shown that in certain information environments, it may be difficult to achieve the optimal result. Under these information environments, if property rights are well defined, then states may be able to negotiate Coase payments to achieve the optimal level of control. While this model applies to many situations, such as sulfur dioxide scrubbers and no-till farming used to reduce nitrogen run-off, there are many other situations that slight variations of this model apply to. Appendix 1.5 demonstrates how the model can be altered to find an optimal policy when reducing a stock-pollutant generates a flow-pollutant. This situation applies to any solid-waste pollutant that can either be stored in a landfill or incinerated.

Several extensions of this research are possible. A simple twist on this model is to examine the situation where a flow pollutant from an upstream state collects as a stock-pollutant in a downstream state. To make this a simple scenario, cleaning up the flow pollutant in the upstream state would not be modeled as creating a stock-pollutant. A model with downstream stock pollution and upstream stock pollution resulting from cleaning up the flow would have two stock variables and only one control variable. This would preclude finding a theoretical solution. Another possible extension is to add another control variable to the model by allowing the firms to vary the amount of emissions they use in production. It would also be possible to introduce uncertainty into this model, either in the damage functions, or in the effectiveness of the control measures. Finally, one could look at particular examples of this problem, specify functional forms, and estimate the parameters of the model in order to find real world optimal tax rates. This could be important for seeing if the optimal tax rate changes significantly over time or if a more feasible constant tax rate can approximate the time path of the optimal tax rate.

Chapter 2: An Intertemporal Computable General Equilibrium Model with Labor and Capital Adjustment Costs

2-1 INTRODUCTION

Adjustment costs can be critical in describing how an economy evolves after a shock or after the implementation of a new policy. Many applied economic models focus on a single period, however, either a short run equilibrium or the long run steady state. These models often ignore transitional dynamics and have no role for adjustment costs. A growing number of computable general equilibrium (CGE) models have included intertemporal behavior and explicitly modeled investment with capital adjustment costs. However, they still generally assume perfect mobility of workers between occupations and industries – despite the empirical evidence demonstrating the importance of labor adjustment costs in firms' dynamic behavior. These types of models are clearly inappropriate for analyzing many types of policies. Particularly problematic for these models are environmental policies that strongly affect a narrow section of the economy. In the long run, the industries adversely affected by such policies will shrink. In the short run, however, the labor employed in those industries is not able to move. Labor adjustment costs prevent miners from quickly becoming computer programmers, for example. The true cost of the policy critically depends on how the economy transitions to the long run equilibrium. While traditional computable general equilibrium models may accurately capture the long run effects of the policy, they miss the important dynamic transition to the long run.

The purpose of this chapter is to present a CGE model that includes both capital and labor adjustment costs in order to capture transitional dynamics and the short-run costs associated with policy proposals in addition to the long run costs. The model presented here is a small general equilibrium model built to demonstrate a method for including labor adjustment costs in a CGE context. The model shows the importance of labor adjustment costs in determining firms' dynamic responses to changes and other shocks. Furthermore, the model demonstrates the need to consider labor adjustment costs in a general equilibrium context. For simplicity, labor adjustment costs are incorporated into one sector only. This formulation can be thought to imply that only one sector requires labor with firm-specific skills. The other sectors employ labor with general skills. A single population of workers may be employed in two different types of jobs: they may have 'salaried' jobs in the sector with labor adjustment costs, or they may have 'hourly' jobs in any sector. The workers with salaried jobs can be thought of as workers that have been trained with firm-specific skills. The workers with hourly jobs, on the other hand, can be considered general-skills workers, meaning that they do not have any firm-specific skills. All workers can choose their level of effort. This is a discrete choice, workers may choose a high level of effort, or they may choose to shirk and exert a low level of effort. The effort level of workers in hourly jobs is perfectly observable, so these workers are unable to get away with shirking. However, the effort level of workers with salaried jobs is not perfectly observable. These workers have the option to shirk, and a chance to get away with it. If they are caught shirking, they loose their salaried job and must find work in an hourly job. The firm employing salaried workers thus has an incentive to pay them an efficiency wage premium to ensure they exert a high level of effort.[9] This simple formulation allows labor adjustment costs to be incorporated without making the model overly complex. Using the methodology presented here, it would be possible to extend the model to include labor adjustment costs in all sectors. The inclusion of labor adjustment costs opens up many other possibilities for CGE models.

Large literatures exist studying both labor adjustment costs[10] and capital adjustment costs[11], but only the latter appear in CGE models. The inclusion of labor adjustment costs in intertemporal CGE models has many potential benefits. Traditionally, CGE models have used four different ways of modeling labor supply. The two simplest methods use either a fixed exogenous labor supply or a fixed exogenous wage rate.[12] In both of these cases, no decision needs to be modeled. Instead, the first case simply represents a 'long run neo-classical' situation with full employment. The fixed exogenous wage rate in the second case is imposed on an otherwise normal labor supply and demand curves, resulting in a slack labor market with unemployment. Third, McKibbin and Sachs (1989) provide a model with a fixed but slowly adjusting wage rate. While this

[9] See Shob (2003) for a survey of the double dividend hypothesis of environmental taxes, including employment and welfare effects in the presence of unemployment, which are sometimes modeled using efficiency wages.

[10] See Hamermesh and Pfann (1996b) for an extensive overview of factor adjustment costs with a focus on labor adjustment costs.

[11] Early examples of investment models with adjustment costs include Gould (1968) and Treadway (1969), Goulder and Summers (1989) is one of the earliest examples of capital adjustment costs in a CGE framework.

[12] See Dixon *et al.* (1988) for an example of a model with a fixed exogenous wage rate.

formulation still does not model a decision process and is not derived from theory, it does capture short run unemployment and the long run adjustment back to full employment. The fourth and last method is to model a fully endogenous labor supply decision. This formulation allows both wages and the level of employment to adjust to shocks. However, full employment is assumed by the model clearing. Furthermore, this formulation does not allow one to model realistic dynamic paths of employment, as the firm will make discrete jumps between employment levels in response to shocks. This chapter introduces a new, fifth type of model, one that includes labor adjustment costs to model an endogenous labor supply decision and examine the trajectory of employment through time.

Section 2-2 of this chapter presents the theoretical model. It also describes in order the sectors of the model: sector A is the consumer goods sector; sector B is the capital services sector; sector C produces raw capital goods; sector D is the intermediate goods sector; and finally, a government sector that collects taxes and gives lump sum subsidies to the entire population. All markets are competitive. This section also details the decisions of the workers in the model's two types of jobs: workers with salaried jobs, or jobs requiring 'firm-specific skills,' are employed in sector A and receive a guaranteed base salary; workers with hourly jobs, or jobs requiring 'general-skills,' can work in all sectors, and firms do not incur adjustment costs when hiring them. The last equations of the theoretical model are the market clearing conditions. Section 2-3 then covers the implementation of the model. The data and parameterization are covered in this section, as is the construction of the steady state computational model. The steady state model is programmed in Ox and is used to provide starting values for the intertemporal model, which is also presented in section 2-3. The intertemporal model is programmed using GEMPACK and covers a 70-year time period. The solution methods used for solving the intertemporal model are also presented here. Procedures for testing the model are the last item covered in this section. Section 2-4 presents simulation results, and section 2-5 concludes.

2-2 THEORETICAL MODEL

This chapter presents a sequence of short run general equilibrium models with four sectors, workers in two types of jobs, and both capital and labor adjustment costs. The model is an extension of Wilcoxen (1989), which presents a small general equilibrium model incorporating investment with capital adjustment costs. Intertemporally, the sequence of models is linked together by a model of both investment adjustment costs and a labor adjustment costs. Sector A produces consumer goods (Q_{net}), sector B produces capital services (K_b), sector C produces raw capital goods (X_k), and sector D produces intermediate goods (X_m). The model includes a population of workers, which are divided into two groups: workers employed in salaried jobs, who have been trained with skills specific to sector A; and workers with hourly jobs, who possess general skills. Only sector A employs workers in salaried jobs. Sector A incurs adjustment costs when it hires or fires workers in salaried jobs, and when it changes the total number of workers in salaried jobs. Workers with hourly jobs are employed in all sectors. Hiring hourly workers does not impose adjustment costs. The model assumes full employment. The model includes two types of capital, K_a, which is created by industry A's investment and used exclusively in sector A, and K_b, which sector B creates with its investment and which it rents out to sectors C and D. Finally, a government is included. The government imposes taxes, gives lump sum subsidies to the entire population, and is constrained to balance its budget. A graphical representation of the production side of the model is presented in Appendix 2.6.

2-2-1 Sector A – Consumer Goods

Industry A produces consumer goods using salaried labor, capital (K_a), and intermediate goods (X_m^a) as inputs. Industry A's capital (K_a) is created by its own investment and requires raw capital inputs (X_k^a) as well as hourly labor inputs (L_2^a) to install the capital.[13] The firm's salaried labor input depends on the number of workers it employs (N_1), and the number of flextime hours each employee works (L_1^a).[14] Each salaried employee must work $\bar{L}_1$ hours per day, any additional hours worked count as flextime hours. [15] Workers in salaried jobs must choose a level of effort (F):

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[13] This specific division of labor between type-1 workers, who work in production, and type-2 workers, who install capital, is useful for estimating the capital adjustment cost parameter.

[14] The flextime hours used in this model are not a literal overtime wage for salaried workers. Instead they are an approximation of a system of bonuses and comp-time normally used to reward salaried workers for working extra hours.

[15] The choice of a guaranteed $\bar{L}_1$-hour workday plus flextime hours for salaried workers instead of a more standard eight-hour day and overtime hours allows for a number of guaranteed hours smaller than eight, thus we refer to additional hours worked (L_1^a) as 'flextime' hours instead of 'overtime' hours. The purpose of this construction is to eliminate the need for a non-negativity

$$
2.1 \qquad F = \begin{Bmatrix} 1 & \\ \text{or} & \\ \sigma & 0 \le \sigma \le 1 \end{Bmatrix} .
$$

The total salaried labor input for the firm is thus $N_1(\overline{L}_1 + L_1^a)F$. As we will discuss later, sector A pays its salaried workers an efficiency wage premium that ensures the workers will choose to exert the high level of effort. Since the salaried workers rationally choose F=1, the F is suppressed in the equations for sector A. A Cobb-Douglas functional form is used for production, so that the firm's gross production function is:

$$
2.2 \qquad f\left(N_1\left(\overline{L}_1 + L_1^a\right), K_a, X_m^a\right) = \varepsilon_a \left(N_1\left(\overline{L}_1 + L_1^a\right)\right)^{\gamma_1^a} (K_a)^{\gamma_2^a} \left(X_m^a\right)^{\gamma_3^a} \ ,
$$

where $\sum_i^3 \gamma_i^a = 1$.

---

constraint and multiple cases that would be involved if we used a standard eight-hour workday with overtime wages. In order to model the overtime labor choice, a non-negativity constraint on overtime labor hours (L) would be required. This introduces two cases into the model, $L_1^a = 0$ and $L_1^a > 0$. If the firm is hit by a severe negative shock, they may want to reduce their output by more than is possible by simply reducing overtime hours. The firm would then enter into the case where $L_1^a = 0$ and reduce its work force. In order to avoid this possibility, we assign a value of 6 to $\overline{L}_1$. With this specification, the firm can be expected to utilize several hours of flextime labor in the steady state. If the firm is subjected to a negative shock, it has much more room to reduce output by reducing flextime labor without running into the constraint that flextime labor must be greater than or equal to zero. In this way the multiple cases are not entirely eliminated; however, the model is formulated so the case where $L_1^a = 0$ is a remote enough possibility that we can ignore it for the vast majority of possible experiments. Another way to deal with this issue is to interpret $L_1^a < 0$ as workers choosing to take unpaid leave.

The firm also faces labor adjustment cost, $C(H_1,N_1)$, which is a function of the hiring rate of salaried workers per period (H_1) and the number of salaried workers the firm currently employs (N_1). The labor adjustment cost is actually a quantity measure of lost output due to the adjustment of labor. The firm loses some of its productivity when it diverts some of its resources to train new employees in the firm-specific skills required for the job, or when it restructures its organization to support a different total number of salaried employees. In a general equilibrium model, labor adjustment costs cannot be simply inserted into a firm's total cost equation. The production function for adjustment costs must be specified. If a firm pays a dollar-value adjustment cost, then some entity must receive those payments. Labor adjustment costs in this model are produced with the same production function as the firm's output. Since the firm's adjustment costs in this model are a quantity measure of lost output instead of a payment, there is no need to create an entity to collect the payments. The firm's gross output is simply its net output from above minus the adjustment costs.

This firm's decision in sector A is broken up into two portions, a within-period decision, and an intertemporal choice. Within each period, the firm's capital stock and the number of workers it employs in salaried jobs are fixed. Thus, the firm minimizes costs within each period by choosing the level of flextime labor and the level of intermediate goods. The labor adjustment costs will act as a fixed cost within each period, since salaried labor stock is fixed in the short run. The capital adjustment costs are represented by a convex adjustment cost function, $C_I^a(I_a)$. These costs are also a short-run fixed cost since investment decisions are made intertemporally. The firm can adjust its capital stock and its number of salaried employees only in the intertemporal decision process, which is represented by an optimal control problem where the firm maximizes dividends it pays out.

The firm's within period cost minimization problem is:

2.3 $$\min_{L_1^a, X_m^a} \quad W\bar{L}_1 N_1 + W_{L1} L_1^a N_1 + P_m X_m^a + C_I^a(I_a)$$
$$\text{s.t. } \varepsilon_a \left(N_1\left(\bar{L}_1 + L_1^a\right)\right)^{\gamma_1^a} (K_a)^{\gamma_2^a} \left(X_m^a\right)^{\gamma_3^a} - C(H_1, N_1) \geq Q_{net} \ .$$

Where $W\bar{L}_1$ is the salary paid to workers trained with firm-specific skills. Salaried workers are compensated at a rate of W_{L1} for the additional flextime hours they work, and the price of intermediate goods is P_m.

The resulting Lagrangian is:

2.4 $$\ell = W\bar{L}_1 N_1 + W_{L1} L_1^a N_1 + P_m X_m^a + C_I^a(I_a)$$
$$-\lambda_a^{sr}\left[\varepsilon_a \left(N_1\left(\bar{L}_1 + L_1^a\right)\right)^{\gamma_1^a} (K_a)^{\gamma_2^a} \left(X_m^a\right)^{\gamma_3^a} - C(H_1, N_1) - Q_{net}\right],$$

where λ_a^{sr} is the Lagrangian multiplier in the short run (sr) for sector A. The cost minimization provides the following first order conditions:

2.5 $$\frac{\partial \ell}{\partial L_1^a} = W_{L1} N_1 - \lambda_a^{sr} \gamma_1^a N_1 \varepsilon_a \left(N_1(\bar{L}_1 + L_1^a)\right)^{\gamma_1^a - 1} (K_a)^{\gamma_2^a} \left(X_m^a\right)^{\gamma_3^a} = 0$$

2.6 $$\frac{\partial \ell}{\partial X_m^a} = P_m - \lambda_a^{sr} \gamma_3^a \varepsilon_a \left(N_1(\bar{L}_1 + L_1^a)\right)^{\gamma_1^a} (K_a)^{\gamma_2^a} \left(X_m^a\right)^{\gamma_3^a - 1} = 0$$

$$2.7 \qquad \frac{\partial \ell}{\partial \lambda_a^{sr}} = \varepsilon_a \left(N_1(\bar{L}_1 + L_1^a)\right)^{\gamma_1^a} (K_a)^{\gamma_2^a} \left(X_m^a\right)^{\gamma_3^a} - C(H_1, N_1) - Q_{net} = 0 .$$

Solving the first order conditions for $X_m{}^a$ results in:

$$2.8 \qquad X_m^a = \left[\frac{1}{\varepsilon_a}\left(Q_{net} + C(H_1, N_1)\right)\right]^{1/(1-\gamma_2^a)} \left(\frac{P_m}{W_{L1}} \frac{\gamma_1^a}{\gamma_3^a}\right)^{-\gamma_1^a/(1-\gamma_2^a)} (K_a)^{-\gamma_2^a/(1-\gamma_2^a)} .$$

Then, solving the first order conditions for $N_1L_1{}^a$, the total amount of flextime labor the firm uses provides:

$$2.9 \qquad N_1 L_1^a = \left[\frac{1}{\varepsilon_a}\left(Q_{net} + C(H_1, N_1)\right)\right]^{\frac{1}{(1-\gamma_2^a)}} \left(\frac{P_m}{W_{L1}} \frac{\gamma_1^a}{\gamma_3^a}\right)^{\frac{\gamma_3^a}{(1-\gamma_2^a)}} (K_a)^{\frac{-\gamma_2^a}{(1-\gamma_2^a)}} - \bar{L}_1 N_1 .$$

Next, we substitute equations 2.8 and 2.9 into total cost and solve for the firm's marginal cost per unit of output:

$$2.10 \qquad MC_a = \frac{1}{\varepsilon_a}\left[\frac{1}{\varepsilon_a K_a}\left(Q_{net} + C(H_1, N_1)\right)\right]^{\frac{\gamma_2^a}{(1-\gamma_2^a)}} \left(\frac{W_{L1}}{\gamma_1^a}\right)^{\frac{\gamma_1^a}{(1-\gamma_2^a)}} \left(\frac{P_m}{\gamma_3^a}\right)^{\frac{\gamma_3^a}{(1-\gamma_2^a)}} .$$

Now impose the assumption that the price of output (P) is equal to marginal cost, and solve for net output:

2.11 $$Q_{net} = \varepsilon_a K_a \left(\varepsilon_a P\right)^{\frac{1-\gamma_2^a}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - C\left(H_1, N_1\right) .$$

Substituting gross output from equation 2.11 into equation 2.8, the factor demand for intermediate goods becomes:

2.12 $$X_m^a = \left(K_a\right)\left(\varepsilon_a P\right)^{\frac{1}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m}\right)^{\frac{1-\gamma_1^a}{\gamma_2^a}} .$$

By substituting gross output from equation 2.11 into equation 2.9, the factor demand for flextime labor becomes:

2.13 $$N_1 L_1^a = \left(K_a\right)\left(\varepsilon_a P\right)^{\frac{1}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{1-\gamma_3^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - \bar{L}_1 N_1 .$$

This completes the firm's within-period problem. Given the firm's stock of capital and number of employees in salaried jobs, as well as the prices, the firm has factor demands for intermediate goods and flextime labor as given in equations 2.12 and 2.13. The firm produces the amount of output shown in equation 2.11.

Now that we know how the firm behaves within each period, we can turn our attention to the firm's intertemporal decision. The firm will use hiring and investment to change its labor and capital stocks intertemporally. When the firm changes its capital stock or hires additional salaried workers, it faces adjustment costs. All new salaried workers the firm hires must be trained before they reach their productive potential, and if the firm changes the size of its workforce, then productivity will be reduced during the reorganization. For capital adjustment costs, we first specify the firm's investment-good production function. We assume a Leontief production function, which requires the firm to purchase raw capital goods (X_k^a) and hire hourly labor (L_2^a) to install it. The firm's resulting investment-good production function is:

2.14 $$I_a = \min\left\{X_k^a, \left(L_2^a/\theta_a\right)^{1/2}\right\}.$$

Notice that the amount of hourly labor required in this sector is proportional to the square of the amount of raw capital -- this will result in convex internal adjustment costs. In order for the firm to double investment, it would need to double its use of raw capital inputs and quadruple its use of hourly labor. Next we minimize investment costs subject to the above production function and find:

2.15 $$I_a = X_k^a = \left(L_2^a/\theta_a\right)^{1/2}.$$

Solving for the firm's raw capital and hourly labor demands we find:

2.16 $$X_k^a = I_a$$

and

2.17 $\quad L_2^a = \theta_a (I_a)^2$.

We can now derive the firm's investment cost function:

2.18 $\quad C(I_a) = P_k I_a + W_{L2}\theta_a (I_a)^2$.

The firm's intertemporal optimal control problem, with H_1 and I_a as the control variables and N_1 and K_a as the state variables, is to choose a time path for investment and hiring of salaried labor that maximizes dividends. The optimal control problem can be written as:

2.19
$$\max_{H_1, I_a} \int_0^\infty [\, P[f(N_1(\overline{L}_1 + L_1^a), K_a, X_m^a) - C(H_1, N_1)] - W\overline{L}_1 N_1 + W_{L1} L_1^a N_1$$
$$- P_m X_m^a - (P_k I_a + W_{L2}\theta_a (I_a)^2)\,](1-\tau_d)e^{-rt}dt$$
$$\text{s.t. } \dot{N}_1 = H_1 - \delta_{n1} N_1 \text{ and } \dot{K}_a = I_a - \delta_{ka} K_a ,$$

where τ_d is the tax rate on dividends, δ_{n1} is the exogenous quit rate of the salaried labor stock, and δ_{ka} is the depreciation rate of the capital stock. Substituting values for the factor demands and gross output from the firm's within period optimization into 2.19, we can rewrite the problem as:

$$2.20 \quad \max_{H_1, I_a} \int_0^\infty \Bigg[P\left[\varepsilon_a(K_a)(\varepsilon_a P)^{\frac{1-\gamma_2^a}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - C(H_1, N_1)\right]$$

$$- N_1(W\bar{L}_1 - W_{L1}\bar{L}_1) - (P_k I_a + W_{L2}\theta_a(I_a)^2)$$

$$- W_{L1}\left[(K_a)(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{1-\gamma_3^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}\right]$$

$$- P_m\left[(K_a)(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{1-\gamma_1^a}{\gamma_2^a}}\right] \Bigg] (1-\tau_d)e^{-rt}dt$$

s.t. $\dot{N}_1 = H_1 - \delta_{n1}N_1$ and $\dot{K}_a = I_a - \delta_{ka}K_a$.

Now we can rewrite the firm's problem as a Hamiltonian, $\hbar$:

$$2.21 \quad \hbar = \Bigg[P\left[\varepsilon_a(K_a)(\varepsilon_a P)^{\frac{1-\gamma_2^a}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - C(H_1, N_1)\right]$$

$$- N_1(W\bar{L}_1 - W_{L1}\bar{L}_1) - (P_k I_a + W_{L2}\theta_a(I_a)^2)$$

$$- W_{L1}\left[(K_a)(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{1-\gamma_3^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}\right]$$

$$- P_m\left[(K_a)(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{1-\gamma_1^a}{\gamma_2^a}}\right] \Bigg] (1-\tau_d)e^{-rt}$$

$$+ \Lambda_{1a}^{lr}[H_1 - \delta_{n1}N_1] + \Lambda_{2a}^{lr}[I_a - \delta_{ka}K_a].$$

The firm's optimization gives us the following first order conditions,

2.22 $$\frac{\partial \hbar}{\partial N_1} = \left[-P\frac{\partial C(H_1, N_1)}{\partial N_1} - W\overline{L}_1 + W_{L1}\overline{L}_1 \right](1-\tau_d)e^{-rt} - \Lambda_{1a}^{lr}\delta_{N1} = -\dot{\Lambda}_{1a}^{lr}$$

2.23 $$\frac{\partial \hbar}{\partial H_1} = -P\frac{\partial C(H_1, N_1)}{\partial H_1}(1-\tau_d)e^{-rt} + \Lambda_{1a}^{lr} = 0$$

2.24 $$\frac{\partial \hbar}{\partial \Lambda_{1a}^{lr}} = H_1 - \delta_{n1}N_1 = \dot{N}_1$$

2.25 $$\frac{\partial \hbar}{\partial K_a} = (\varepsilon_a P)^{\frac{1}{\gamma_2^a}}[\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - W_f\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{1-\gamma_3^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}$$
$$- P_m\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{1-\gamma_1^a}{\gamma_2^a}}](1-\tau_d)e^{-rt} - \Lambda_{2a}^{lr}\delta_{ka} = -\dot{\Lambda}_{2a}^{lr}$$

2.26 $$\frac{\partial \hbar}{\partial I_a} = [-P_k - 2W_{L2}\theta_a I_a](1-\tau_d)e^{-rt} + \Lambda_{2a}^{lr} = 0$$

2.27 $$\frac{\partial \hbar}{\partial \Lambda_{2a}^{lr}} = I_a - \delta_{ka}K_a = \dot{K}_a \,.$$

Since this is an infinite horizon problem, we can make a convenient transformation and define λ_i such that $\Lambda_i = \lambda_i e^{-rt}$, where Λ_i is the present value multiplier, and where λ_i is the current value multiplier. The present value multiplier, Λ_i , is the change in value at time t = 0 from having an extra unit of capital or an extra salaried employee at time t. The current value multiplier, λ_i , is the change in value in year t from a small change in the stock of capital or number of employees in year t. Using this transformation and rearranging the first order conditions results in the following equations:

2.28 $\lambda_{1a}^{lr} = P\frac{\partial C(H_1, N_1)}{\partial H_1}(1-\tau_d)$

2.29 $\dot{\lambda}_{1a}^{lr} = (r+\delta_{n1})\lambda_{1a}^{lr} + \left[P\frac{\partial C(H_1, N_1)}{\partial N_1} + \left[W\bar{L}_1 - W_{L1}\bar{L}_1\right]\right](1-\tau_d)$

2.30 $\dot{N}_1 = H_1 - \delta_{n1}N_1$

2.31 $\lambda_{2a}^{lr} = \left[P_k + 2W_{L2}\theta_a I_a\right](1-\tau_d)$

2.32 $\dot{\lambda}_{2a}^{lr} = (r+\delta_{ka})\lambda_{2a}^{lr} - \left[\gamma_2(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}\right](1-\tau_d)$

2.33 $\dot{K}_a = I_a - \delta_{ka}K_a$.

Equations 2.30 and 2.33 are the equations of motion for the number of salaried employees and the capital stock respectively. Using the rearranged first order conditions, we can solve for the equations of motion for hiring and investment. Given the differentiability of equations 2.28 and 2.31, the equations of motion are:

2.34 $\dot{H}_1 = \dfrac{(r+\delta_{n1})\frac{\partial C}{\partial H_1} + \frac{\partial C}{\partial N_1} + \left[W\bar{L}_1 - W_{L1}\bar{L}_1\right] - \frac{\partial^2 C}{\partial H_1 \partial N_1}(H_a - \delta_{n1}N_1)}{\partial^2 C / \partial H_1^2}$

2.35 $\dot{I}_a = \dfrac{(r+\delta_k)\left[P_k + 2W_{L2}\theta_a I_a\right] - \gamma_2(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}}{2W_{L2}\theta_a}$.

In order to do more with this model, we now must assume a functional form for the labor adjustment costs, $C(H_1,N_1)$. The literature on labor adjustment costs includes debates about several different possible functional forms. Hamermesh and Pfann (1996b) discuss the merits and drawbacks of several different functional forms used for labor adjustment costs. In partial equilibrium models, the most prominent form for labor adjustment costs is symmetric convex (quadratic) costs. This functional form has been used extensively because it provides a decent first order approximation and is simple to work with. The imposition of symmetry, however, has never been highly regarded, as negative net employment changes have generally been found to be cheaper than positive net adjustments (Hamermesh and Pfann, 1996a). Another approach to modeling labor adjustment costs is to use asymmetric convex adjustment costs. Pfann and Verspagen (1989) proposed a form of asymmetric convex adjustment costs that has proven to be useful for estimation purposes. Their functional form allows for the direction of the asymmetry to be estimated. Unfortunately, this functional form is complex enough to make its use in our model prohibitively difficult. Another way of modeling adjustment costs is to introduce lumpy adjustment costs. Hamermesh (1989) finds that the standard model of convex labor adjustment costs is inferior to a specification using lumpy adjustment costs in certain situations. Lumpy adjustment costs require using multiple cases to model the decision process. We have found that for programming purposes, including multiple cases in an intertemporal is not feasible when using the GEMPACK modeling package as we do in this chapter. Another important factor in modeling labor adjustment costs, emphasized by Hamermesh (1995), is the distinction between gross and net adjustment costs. Gross labor adjustment costs arise when any worker is hired or fired. Net adjustment costs, on the other hand, are incurred whenever the firm changes the total number of workers employed. This distinction is potentially very important here, as one would expect to see gross adjustment costs occurring at all times if firms face regular turnover, while net adjustment costs would be zero in the steady state and only be incurred when the firm adjusts its stock of employees.

This chapter uses the following convex labor adjustment cost function:

2.36 $$C(H_1, N_1) = \alpha_1 (H_1 - \delta_{n1} N_1)^2 + \alpha_2 (H_1)^2 \ .$$

As Hamermesh (1995) encourages, this labor adjustment cost function contains both gross and net labor adjustment costs. The α_2 term captures gross adjustment costs, which are incurred whenever a salaried employee is hired or fired. Net adjustment costs are incurred when the firm increases or decreases its stock of salaried employees. These costs are captured by the α_1 term.

Turning back to our equations of motion, we can now use the partial derivatives of our adjustment cost function and substitute them into the equation of motion for hiring (2.34). This substitution results in the following rewritten equation of motion:

2.37 $$\dot{H}_1 = \frac{(r + \delta_{n1})(2\alpha_1 (H_1 - \delta_{n1} N_1) + 2\alpha_2 H_1) + \frac{1}{P}\left[W\bar{L}_1 - W_{L1}\bar{L}_1\right]}{2(\alpha_1 + \alpha_2)} \ .$$

In Appendix 2.1, we show that both the capital and salaried labor systems in sector A are saddle path stable. Knowing that the system has a unique saddle path allows us to use the transversality conditions – imposing that the model approaches the steady state in the long run – to provide the necessary end-point boundary conditions.

We can now go back to the equations of motion and solve for the steady state values. By setting the four equations of motion for H_1 dot, N_1 dot, I_a dot, and K_a dot (2.37, 2.30, 2.35 and 2.33 respectively) equal to zero, and solving the system of equations, we can find the unique steady state values of H_1, N_1, I_a, and K_a:

2.38 $$H_1^{ss} = \frac{\frac{1}{P}\left[W_{L1}\bar{L}_1 - W\bar{L}_1\right]}{(r+\delta_{n1})2\alpha_2}$$

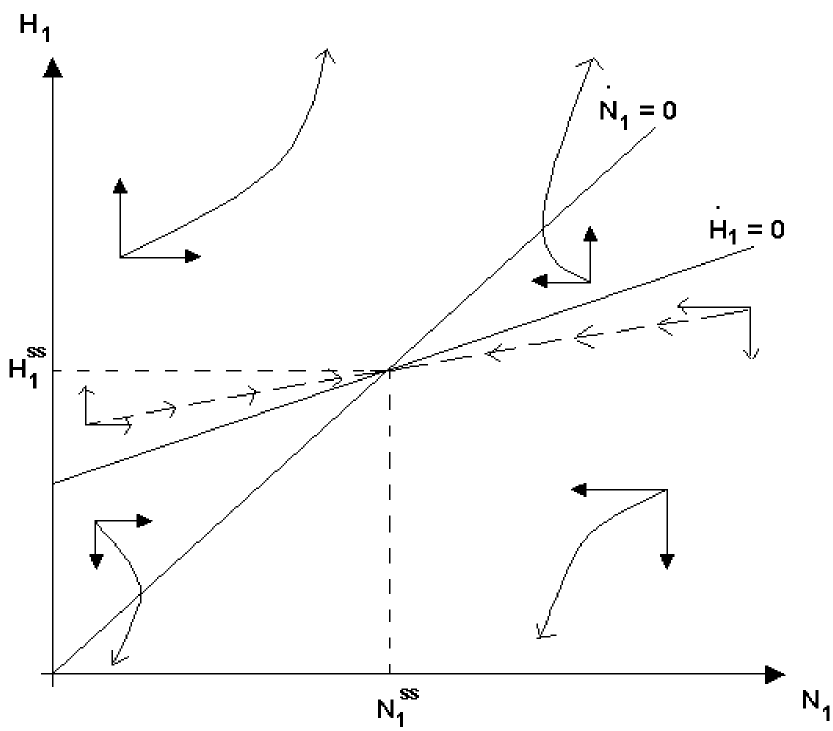
2.39 $$N_1^{ss} = \frac{\frac{1}{P}\left[W_{L1}\bar{L}_1 - W\bar{L}_1\right]}{\delta_{n1}(r+\delta_{n1})2\alpha_2}$$

2.40 $$I_a^{ss} = \frac{\frac{\gamma_2}{(r+\delta_{ka})}(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - P_k}{2W_{L2}\theta_a}$$

2.41 $$K_a^{ss} = \frac{\frac{\gamma_2}{(r+\delta_{ka})}(\varepsilon_a P)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - P_k}{\delta_{ka}2W_{L2}\theta_a}.$$

This system of equations is best understood by looking at phase diagrams representing the firm's investment and hiring practices. Ideally, a single four dimensional phase diagram with H_1, N_1, I_a, and K_a axes could completely depict the model's equations of motion. Since H_1 and N_1, do not appear in the I_a and K_a equations of motion and vice versa, we can present two separate phase diagrams for simplicity. Below is the firm's H_1-N_1 phase diagram:[16]

Figure 2.1 Phase Diagram for Hiring (H_1) and Number of Salaried Jobs (N_1)

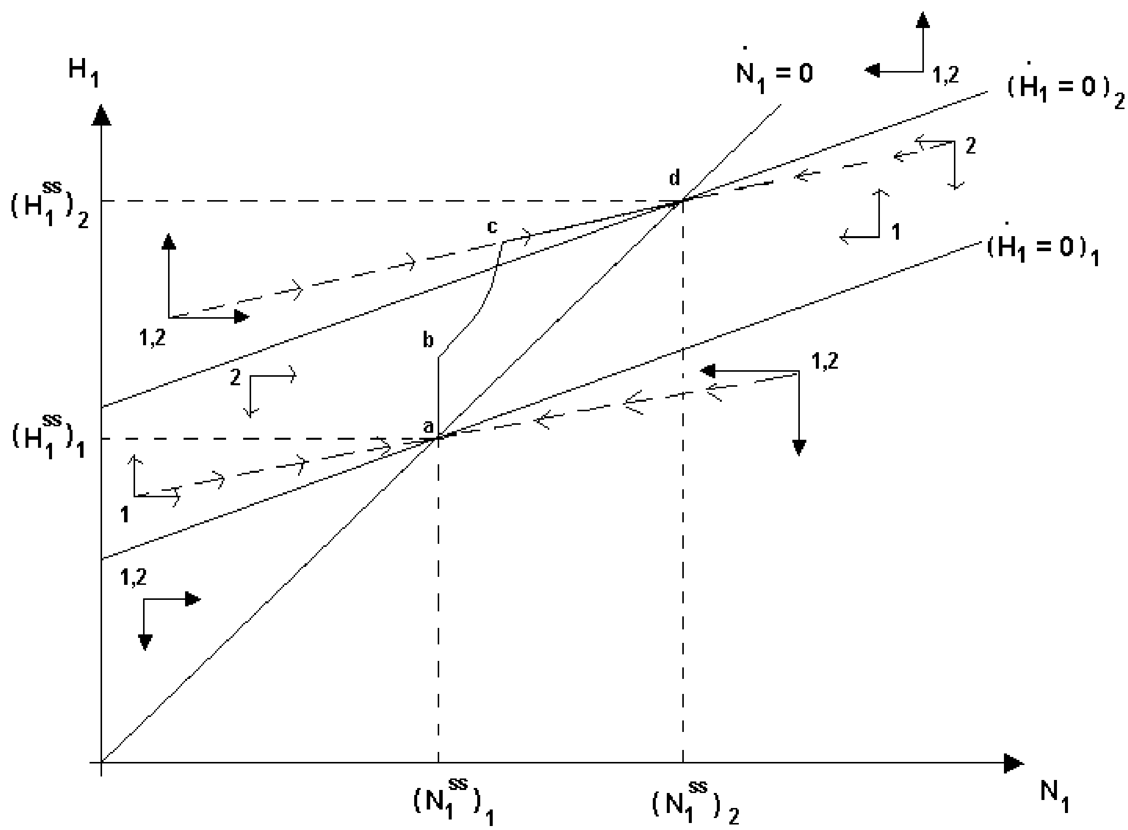


[16] The derivations of the model's phase diagrams are shown in Appendix 2.2.

The saddle path depicts how the stock of salaried employees and the hiring rate evolve over time. If the firm has too many salaried employees, then the hiring rate will be below the $\dot{H}_1 = 0$ isocline and the stock of salaried employees will be below the $\dot{N}_1 = 0$ isocline, thus both the hiring rate and the stock of salaried employees will decrease over time and move towards the steady state equilibrium.

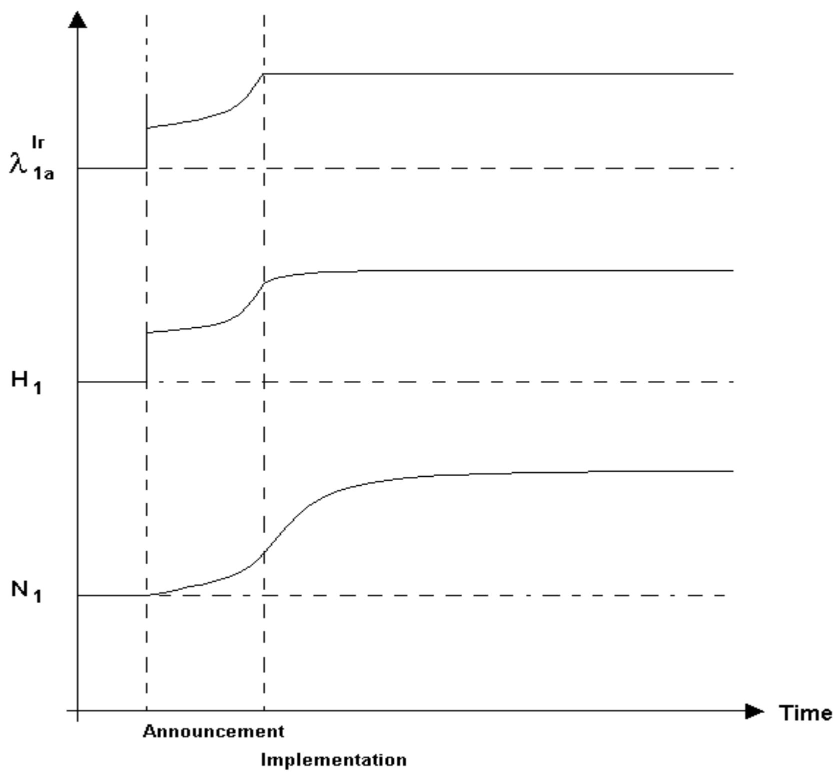
We can now use the phase diagrams to see how the firm responds to exogenous shocks in a partial equilibrium setting. The phase diagram in Figure 2.2 shows the results of an announced permanent decrease of the tax rate on W (wage rate implied by the salary $W\overline{L}_1$) for salaried workers. The wage taxes in this model nominally fall on the workers, and thus do not appear in any equations for sector A; however, the economic incidence of the tax is on both workers and the firm. For the purpose of the phase diagram experiment, we can represent the tax cut, *ceteris paribus*, as a reduction of the implied wage rate for salaried workers. In this experiment, the tax on W is not intended to represent a statutory tax rate, since the tax on W and the tax on hourly wages do not differ. Instead the decrease of the tax on W in this experiment is an explanatory device to see what happens when the taxes do differ (e.g. no taxes on fringe benefits for salaried workers).

Figure 2.2 The Effects of an Announced Decrease of τ_w



In the phase diagram above, the firm begins at the steady state equilibrium shown by point a. As soon as the announcement is made that there will be decrease in the tax rate on salaried employees, the firm increases the hiring rate and jumps to point b. In the period between the announcement of the policy and the implementation of the policy, the firm's hiring rate and stock of salaried employees evolves along a path dictated by the original isoclines. This path is shown as the curve between point b and point c, where both the hiring rate and the stock of employees are increasing. When the new policy is actually implemented and the tax rate is lowered, the $\dot{H}_1 = 0$ isocline shifts up from $(\dot{H}_1 = 0)_1$ to $(\dot{H}_1 = 0)_2$, and the evolution of the hiring rate and the salaried labor stock is governed by the new isoclines. The firm optimally chooses a path such that as soon as the new policy is implemented, it finds itself on the new saddle path. The firm's hiring rate and number of salaried workers then follow the new saddle path towards the new steady state equilibrium. This is shown as the movement between point c and point d in Figure 2.2. The integral curves for the evolution of the state variable (N_1), the control variable (H_1), and the costate variable (λ_{1a}^{lr}) are shown in Figure 2.3. The integral curves say something about the speed of the adjustments shown in the phase diagram. In Figure 2.3, the announcement corresponds to the jump from point a to point b in Figure 2.2. As soon as the policy is announced, the firm increases its hiring rate and the costate variable has a discrete jump to higher level. The implementation of the policy in Figure 2.3 corresponds to point c in the phase diagram. In the period between the announcement and implementation of the policy, both the control variable and the costate variable increase at an increasing rate. The number of salaried workers employed by the firm begins to rise at an increasing rate after the announcement of the new policy. When the new policy is actually implemented, the costate variable reaches its new steady state value. In the phase diagram, Figure 2.2, the model approaches the new steady state along the new saddle path between point c and point d. This happens over a long period of time and corresponds to the period after the implementation as time becomes very large in Figure 2.3. The hiring rate continues to rise after the implementation, but at a much slower rate as it approaches the new steady state. The state variable's integral curve goes through an inflection point when the policy is implemented, and increases at a decreasing rate as it approaches its new steady state value.

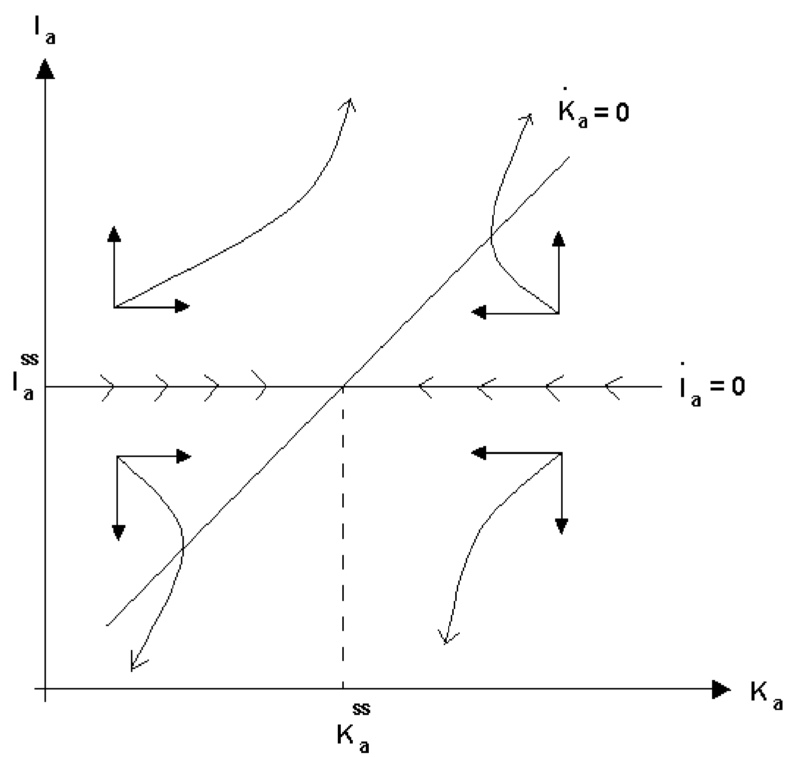
Figure 2.3 Integral Curves for Announced Decrease of τ_w Experiment



This sort of experiment shows the importance of the intertemporal perspective.[17] If attention were limited to the steady state changes in the long run, a lot of important detail in how the steady state value is reached would be missed.

Also of interest is the phase diagram governing the firm's investment decision. Figure 2.4 shows the firm's I_a-K_a phase diagram.

Figure 2.4 Phase Diagram for Investment (I_a) and Capital (K_a)



[17] Appendix 2.3 shows the versatility of this type of partial equilibrium phase diagram analysis by showing the same experiment when the announced policy is only temporary.

This phase diagram shows that the $\dot{I}_a = 0$ isocline has a zero slope; therefore, if the capital stock is too small or large, the firm simply chooses the steady state level of investment and lets the capital stock grow or decay to the optimal size.

2-2-2 Sector B – Capital Services

Industry B produces capital services (K_b), all of which it rents out to sectors C and D at a price, ρ, which it takes as given. As in sector A, all of the firm's profits are paid out as dividends. Capital services are produced by the firm's own investment and require raw capital inputs (X_k^b) as well as hourly labor inputs (L_2^b). As with sector A, we need to specify the firm's investment-good production function. We again assume a Leontief production function:

$$2.42 \qquad I_b = \min\left\{X_k^b, \left(L_2^b/\theta_b\right)^{1/2}\right\}.$$

As with investment in sector A, the amount of hourly labor required is proportional to the square of the amount of raw capital, this will again result in internal capital adjustment costs. Next, minimize investment costs subject to the above production function to find:

$$2.43 \qquad I_b = X_k^b = \left(L_2^b/\theta_b\right)^{1/2}.$$

Solving for the firm's raw capital and hourly labor demands:

2.44 $X_k^b = I_b$

and

2.45 $L_2^b = \theta_b (I_b)^2$.

We can now write down the Hamiltonian corresponding to the firm's maximization problem as:

2.46 $\hbar = \left[\rho K_b - (P_k I_b + W_{L2}\theta_b (I_b)^2)\right](1 - t_d) e^{-rt} + \Lambda_b^{lr}\left[I_b - \delta_{kb} K_b\right]$.

Using the present value multiplier instead of the current value multiplier, the first order conditions resulting from the firm's profit maximization are:

2.47 $\lambda_b = (P_k + W_{L2}\theta_b I_b)(1 - \tau_d)$

2.48 $\dot{\lambda}_b = (r + \delta_{kb})\lambda_b - \rho(1 - \tau_d)$

2.49 $\dot{K}_b = I_b - \delta_{kb} K_b$.

Equation 2.49 is the equation of motion for the capital stock, the equations of motion for investment is:

2.50 $\dot{I}_b = \dfrac{(r + \delta_{kb})(P_k + W_{L2}\theta_b I_b) - \rho}{2 W_{L2}\theta_b}$.

This system is shown to be saddle path stable in Appendix 2.1. The steady state values for industry B's investment and capital stock are:

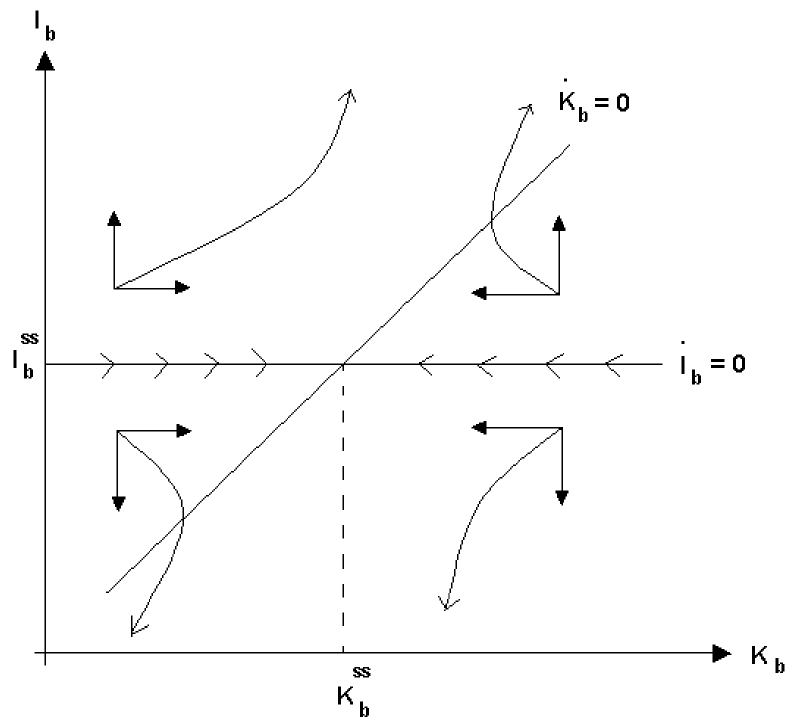
2.51 $$I_b^{ss} = \frac{1}{2W_{L2}\theta_b}\left[\frac{\rho}{(r+\delta_{kb})} - P_k\right]$$

2.52 $$K_b^{ss} = \frac{1}{\delta_{kb}2W_{L2}\theta_b}\left[\frac{\rho}{(r+\delta_{kb})} - P_k\right].$$

Figure 2.5 shows the phase diagram for capital accumulation in sector B:[18]

[18] Derivation of this phase diagram is shown in Appendix 2.2.

Figure 2.5 Phase Diagram for Investment (I_b) and Capital (K_b)



The phase diagram for capital accumulation in sector B is very similar to the I_a-K_a phase diagram for sector A.

2-2-3 Sector C – Raw Capital

Sector C produces the raw capital (X_k) used in sectors A and B for investment. The inputs used in sector C include capital services (K_b^c), which are rented from sector B at the price ρ, and hourly labor (L_2^c), which is paid the wage rate W_{L2}. Since industry C faces no labor adjustment costs and does not invest, it does not have an intertemporal decision. Sector C is a traditional sector that earns no short run profits. Raw capital (X_k) is produced through a Cobb-Douglas production function:

$$2.53 \qquad X_k = \varepsilon_c (L_2^c)^{\gamma_c} (K_b^c)^{1-\gamma_c} \ .$$

The firm's cost-minimization problem leads to the following factor demands:

$$2.54 \qquad K_b^c = \frac{1}{\varepsilon_c} X_k \left(\frac{W_{L2}(1-\gamma_c)}{\rho\gamma_c} \right)^{\gamma_c}$$

$$2.55 \qquad L_2^c = \frac{1}{\varepsilon_c} X_k \left(\frac{\rho\gamma_c}{W_{L2}(1-\gamma_c)} \right)^{1-\gamma_c} \ .$$

The firm is constrained to earn zero profits:

$$2.56 \qquad P_k X_k = (W_{L2} L_2^c + \rho K_b^c)(1+\tau_{xk}) \ ,$$

where τ_{xk} is a sales tax on raw capital goods and P_k is the purchaser's price.

2-2-4 Sector D – Intermediate Goods

The intermediate goods (X_m), used in the production of consumer goods in sector A, are produced in sector D, which is structurally identical to sector C. Intermediate goods are produced according to the following Cobb-Douglas production function:

$$2.57 \qquad X_m = \varepsilon_d (L_2^d)^{\gamma_d} (K_b^d)^{1-\gamma_d} .$$

As with sector C, the firm's cost-minimization problem leads to the following factor demands:

$$2.58 \qquad K_b^d = \frac{1}{\varepsilon_d} X_m \left(\frac{W_{L2}(1-\gamma_d)}{\rho\gamma_d} \right)^{\gamma_d}$$

$$2.59 \qquad L_2^d = \frac{1}{\varepsilon_d} X_m \left(\frac{\rho\gamma_d}{W_{L2}(1-\gamma_d)} \right)^{1-\gamma_d} .$$

The firm is also constrained to earn zero profits:

$$2.60 \qquad P_m X_m = (W_{L2} L_2^d + \rho K_b^d)(1+\tau_{xm}) ,$$

where τ_{xm} is a sales tax on intermediate goods and P_m is the purchaser's price of intermediate goods.

2-2-5 Consumers and Workers

The population of this model (Pop) is divided into workers employed in two types of jobs: workers with salaried jobs are employed in sector A and are trained with firm-specific skills; and workers with hourly jobs, who possess general skills and can be employed in any sector. Workers in salaried jobs can shirk and have a chance of not being caught. If they choose to shirk, then they receive slightly more utility than they would have from working hard. A worker's job is not a permanent characteristic. Workers with hourly jobs can be hired by sector A and trained with firm specific skills to become salaried employees. Alternatively, salaried workers with firm specific skills can leave sector A to use their general skills as hourly workers in the other sectors. This would happen if the salaried worker is caught shirking and is fired from the salaried job. Workers are modeled as having identical Cobb-Douglas preferences between consumption and leisure. The entire population owns equal shares of the two capital goods, so both types of workers receive equal shares of the dividend payments from sectors A and B. Finally, both types of workers receive a subsidy payment from the government (S_1 and S_2).

2-2-5-1 Workers with Salaried Jobs

The utility maximization problem for salaried people is as follows:

2.61 $$\max_{Q_1, J_1} \quad U_1 = g(F)(Q_1)^{\mu}(J_1)^{1-\mu}$$

s.t.

$$P(1+\tau_q)Q_1 = \frac{W(1-\tau_w)\overline{L}_1 + W_{L1}(1-\tau_{L1})(E_1 - J_1) + S_1 + (D_a + D_b)(1-\tau_d)}{Pop},$$

where U_1 is utility, Q_1 is consumption, J_1 is leisure, F is the level of effort, E_1 is the time endowment excluding the standard $\overline{L}_1$ hours of work, D_a and D_b are gross dividend payments, τ_q is a sales tax on consumer goods, τ_w is an equivalent hourly wage tax on workers' salaries, τ_{L1} is the tax on additional flextime labor[19], and τ_d is the dividend tax. Utility depends on the level of effort chosen by the worker. If the worker choose the high level of effort (F = 1), then g(1) = 1. If the worker shirks (F = σ), then $g(\sigma) > 1$.[20] We now define full income (Y_1):

$$2.62 \qquad Y_1 \equiv W(1-\tau_w)\overline{L}_1 + W_{L1}(1-\tau_{L1})E_1 + S_1 + \frac{(D_a + D_b)(1-\tau_d)}{Pop} \ .$$

Performing the consumer's maximization and utilizing the full income definition, we find the following demand equations for consumption and leisure:

$$2.63 \qquad P(1-\tau_q)Q_1 = \mu Y_1$$

$$2.64 \qquad W_{L1}(1-\tau_{L1})J_1 = (1-\mu)Y_1 \ .$$

Finally, salaried workers' flextime labor supply is:

$$2.65 \qquad W_{L1}(1-\tau_{L1})L_1^s = \mu Y_1 - W(1-\tau_w)\overline{L}_1 - S_1 - \frac{(D_a + D_b)(1-\tau_d)}{Pop} \ ,$$

where L_1^s is a salaried worker's flextime labor supply, that is the additional labor supply beyond the first $\overline{L}_1$ salaried hours.

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[19] τ_{L1} is set to zero in this model. This is to represent the fact that the comp-time the flextime hours are designed to represent is generally untaxed. If we interpret these as bonus payments instead of comp-time, then a positive value for τ_{L1} may be desired.
[20] In this chapter we set $g(\sigma) = 1.01$.

2-2-5-2 Workers with Hourly Jobs

Since a worker's job is not a permanent characteristic, the preferences of the two types of workers are identical. The only difference between the two types is their source of income. For workers with hourly jobs, their utility maximization problem is:

$$2.66 \quad \max_{Q_2, J_2} \quad U_2 = (Q_2)^{\mu}(J_2)^{1-\mu}$$

$$\text{s.t.} \quad P(1+\tau_q)Q_2 = W_{L2}(1-\tau_{L2})(E_2 - J_2) + S_2 + \frac{(D_a + D_b)(1-\tau_d)}{Pop}.$$

The variables are defined similarly to those from the salaried worker problem. The exception is that for hourly workers, since they are not guaranteed any hours of work, E_2 is their full time endowment and is thus equal to E_1 plus $\overline{L}_1$ additional hours. Note that g(F) does not appear in the utility function for workers with hourly jobs. This is because their effort level is perfectly observable and they are unable to shirk. The full income definition also differs for workers with hourly jobs and is defined as:

$$2.67 \quad Y_2 \equiv W_{L2}(1-\tau_{L2})E_2 + S_1 + \frac{(D_a + D_b)(1-\tau_d)}{Pop}$$

The consumer's utility maximization problem, along with the full income definition, provides the demand equations for consumption and leisure:

2.68 $P(1-\tau_q)Q_2 = \mu Y_2$

2.69 $W_{L2}(1-\tau_{L2})J_2 = (1-\mu)Y_2$

Also, the labor supply function for hourly workers is:

2.70 $$W_{L2}(1-\tau_{L2})L_2^s = \mu Y_2 - S_2 - \frac{(D_a + D_b)(1-\tau_d)}{Pop} ,$$

where L_2^s is an hourly worker's labor supply.

2-2-6 Government

The government collects tax revenues and gives lump sum payments to the population. Total government expenditures (G) are endogenous and are required to be equal to government revenues:

2.71 $$G = \tau_w W\overline{L}_1 N_1 + \tau_{L1} W_{L1} L_1^s N_1 + \tau_{L2} W_{L2} L_2^s N_2 + \tau_d (D_a + D_b) + \tau_q P(Q_1 N_1 + Q_2 N_2) + \tau_{xk} P_k X_k + \tau_{xm} P_m X_m ,$$

where N_2 is equal to the total population (Pop) less the number of salaried workers (N_1). Government expenditures are divided between the lump sum payments to the two types of workers:

2.72 $G = S_1 N_1 + S_2 N_2$.

Finally, we require that subsidy payments to all individuals be equal:

$$2.73 \qquad S_1 = S_2 .$$

This assumption could be changed for experiments involving unequal subsidies.

2-2-7 Efficiency Wage Condition

Sector A must pay its salaried workers an efficiency wage premium to ensure that they choose to exert a high level of effort. If the salaried workers shirk: they are less productive for the firm; they receive a higher level of utility; and, they have a probability of being caught equal to ψ. If they are caught shirking, then they loose their salaried job forever, and will instead find a job as an hourly worker. In order to ensure that its workers exert a high level of effort, the firm simply needs to pay them enough so that their expected utility for working hard is equal to or higher than, but chosen to be equal to, their expected utility from shirking:

$$2.74 \qquad \int_0^\infty (Q_1)^\mu (J_1)^{1-\mu} e^{-rt} dt = \int_0^\infty \left[(1-\psi^t) g(\sigma)(Q_1)^\mu (J_1)^{1-\mu} + \psi^t (Q_2)^\mu (J_2)^{1-\mu} \right] e^{-rt} dt \ .$$

Solving the above integrals, we find the following condition:[21]

[21] Note that if the probability of being caught (ψ) is one, then this condition simply states that the utility levels for workers in the two different jobs are equal.

$$2.75 \qquad \frac{(Q_1)^{\mu}(J_1)^{1-\mu}}{r} = \frac{\ln(\psi)g(\sigma)(Q_1)^{\mu}(J_1)^{1-\mu}}{-r(r-\ln(\psi))} + \frac{(Q_2)^{\mu}(J_2)^{1-\mu}}{r-\ln(\psi)}.$$

This condition ensures that salaried workers will not shirk and risk losing their jobs.

The last condition we need for this story is one that says that the firm is best responding by offering the efficiency wage premium. This will be the case so long as σ is small enough that the profits the firm would receive from paying a normal salary and allowing salaried workers to shirk, are less than the profits the firm would receive from paying an efficiency wage premium and ensuring that salaried workers exert a high level of effort.

2-2-8 Market Clearing Conditions

Completing the model are a set of market clearing conditions for the consumption good, both types of labor, intermediate goods, raw capital, and capital services:

2.76 $\quad Q_{net} = Q_1 N_1 + Q_2 N_2$

2.77 $\quad L_1^s = L_1$

2.78 $\quad L_2^s N_2 = L_2^a + L_2^b + L_2^c + L_2^d$

2.79 $\quad X_m = X_m^a$

2.80 $\quad X_k = X_k^a + X_k^b$

2.81 $\quad K_b = K_b^c + K_b^d$.

The model is now complete and we can turn to its implementation.

2-3 IMPLEMENTATION

2-3-1 Data and Parameterization

In order to implement the model the variables are partitioned into endogenous and exogenous sets.[22] Once the model has been partitioned, we must find values for the exogenous variables and the parameters of the model.[23] Values for the exogenous variables are taken from a variety of sources.[24] The capital depreciation rate parameters for sectors A and B (δ_a and δ_b) are set to 10%. The exogenous quit rate (δ_{n1}) is set to 5% (Jaramillo *et al.*, 1993). Three data sets are used to estimate the parameters on the production side of the model. The primary

[22] See Table 2.6 for the list of exogenous variables and Table 2.7 for the list of endogenous variables.

[23] Most of the parameters of the model are estimated, however some are assumed. It should also be noted that the choice of Cobb-Douglas functional forms imposes unitary elasticity of substitution, own-price elasticity equal to one, and cross-price elasticity equal to zero.

[24] The price of the consumption good is chosen to be the numeraire price and simply set to one. The required number of hours worked ($\overline{L}_1$) is chosen to be six hours. The time endowment for hourly workers (E_2) is chosen to be 16 hours, thus the time endowment for salaried workers excluding required hours (E_1) is 10 hours. The interest rate (r) is chosen to be 5%. The probability of a salaried worker being caught shirking (ψ) is set to 25%. The utility multiplier for shirking ($g(\sigma)$) is chosen to be 1.01,. The population (Pop) is taken from data on the total number of worker jobs form the US Department of Labor - Bureau of Labor Statistics - Special Purpose Files – Industry Employment and Output. The tax on implied wages for salaried workers and the hourly wage tax, (τ_W, τ_{L2}) were set equal to the US average marginal federal income tax rate, and the dividend tax was set equal to the US average marginal federal income tax rate on dividends (Feenberg & Coutts, 1993). The tax on flextime wages, τ_{L1}, was set equal to zero. The sales tax (τ_S), tax on intermediate goods (τ_{xm}), and tax on raw capital (τ_{xk}) were simply set equal to 5%.

data set is Jorgenson's 35-industry input-output (I-O) table (Jorgenson, 2003).[25] This data set provides most of the information needed to estimate the parameters of the model covering the years 1947 - 1985. However, this I-O table only has information on one labor type. Data on different labor types comes from the US Department of Labor - Bureau of Labor Statistics - Special Purpose Files – Industry Employment and Output. This data set provides information for 192 different industries on the number of jobs and total hours worked for three types of workers: production workers, wage & salary workers, and self-employed & unpaid family workers. This data is available from the years 1958 – 2000; however it is very incomplete before the year 1975. We still need information on wages to perform our estimations, so the US Department of Labor - Bureau of Labor Statistics - Current Employment Statistics are used as the final data set. The Current Employment Statistics have information of wages of production workers by Standard Industrial Classification (SIC) code.[26] This data set is also available from the years 1958 – 2000, and is rather incomplete before 1972.

The first step in merging the three data sets is to merge the two BLS data sets on SIC code. Once these two data sets are merged, they need to be aggregated up from the initial 192-order industries to the 35-order industries contained in the I-O table. In order to perform this aggregation, a mapping between the 192-order industries of the BLS data set and the 35-order industries

[25] Descriptions of the 35 industries along with a mapping to the 4 sectors of this model are presented in Table 2.3.

[26] The BLS Special Purpose Files – Industry Employment and Output provide an industry sector plan that maps their 192 industries into the corresponding SIC code (ftp://ftp.bls.gov/pub/special.requests/ep/ind.employment/sect281.txt).

of the I-O table must be created. This mapping is presented in Table 2.2. Once the mapping has been created, the number of jobs and total hours worked for the various labor types in each of the 35-order industries can be found by simple summation. We create a price index for production workers' wages in each of the 35-order industries using divisia aggregation (Jorgenson and Griliches, 1971). Total hours and jobs for the three labor types and production worker wages can then be merged with the I-O table data set on 35-order industry.

With the three data sets now merged, we need to aggregate up from 35 industries to the 4 sectors of this model. A mapping from the 35-order industries to the 4 sectors of this model is presented in Table 2.3. Roughly, services and non-durable consumption goods are mapped to sector A, construction and all types of durable goods are mapped to sector C, and all other goods, mainly non-consumption non-durables and services, are mapped to sector D. No industries are mapped to sector B, this is because the capital services sector in this model simply nails down the raw capital produced in other sectors. The values and quantities are aggregated by simple summation, and the prices and wages are aggregated by again using divisia aggregation to create price indicies (Jorgenson and Griliches, 1971). Finally, two hiring variables are generated for production workers. With data on the number of production worker jobs in successive periods, we create a variable for net hiring of production workers by simply subtracting the number of production worker jobs in the previous period from the number of production worker jobs in the current period. With the assumption of a 5% exogenous quit rate, the gross number of production workers hired in a period is set equal to the total number of production worker jobs in the current period less 95% of the total number of production worker jobs in the previous period.

The majority of the production parameters are estimated simultaneously using nonlinear least squares. The first two parameters estimated in the simultaneous regression are γ_c and γ_d. The technical change parameters for sectors C and D (ε_c and ε_d) are the next two parameters estimated in the simultaneous regression. The investment parameter for sectors A, θ_a, is also estimated here.[27] One of the Cobb-Douglas production parameters from sector A, γ_3, and the ratio of γ_1/γ_3 are estimated in the simultaneous regression.[28] The last parameter estimated here is the technical change parameter (ε_a) for sector A. This simultaneous regression is preformed using the merged data set from 1975-1985.

There are seven equations included in the simultaneous regression. From the first order conditions of the profit maximization problems for sectors C and D, we can find the following expressions for γ_c and γ_d:[29]

2.82 $$\gamma_c = \frac{W_{L2} L_2^c}{W_{L2} L_2^c + \rho K_b^c}$$

2.83 $$\gamma_d = \frac{W_{L2} L_2^d}{W_{L2} L_2^d + \rho K_b^d}.$$

[27] In the mapping from the 35-order industries to the 4 sectors of the model, none of the 35-order industries are mapped to sector B. Thus, θ_b is set equal to θ_a.

[28] From these two estimated parameters and the fact that γ_1, γ_2, and γ_3 sum to one allows us to solve for all three Cobb-Douglas production parameters in sector A.

[29] For sectors B, C, and D all of the labor from the I-0 table is assumed to be hourly labor, the only type used by these sectors in the model.

Taking logs of the raw capital and intermediate goods production functions, we get the following equations:

2.84 $\ln(X_k) = \ln(\varepsilon_c) + \gamma_c \ln(L_2^c) + (1-\gamma_c)\ln(K_b^c)$

2.85 $\ln(X_m) = \ln(\varepsilon_d) + \gamma_d \ln(L_2^d) + (1-\gamma_d)\ln(K_b^d)$.

From equation 2.15, we can solve for θ_a as a function of hourly labor and raw capital.[30]

2.86 $\theta_a = \dfrac{L_2^a}{(X_k^a)^2}$.

From first order conditions of the firm's within period cost minimization problem in sector A, we can find the ratio γ_1/γ_3:

2.87 $\Gamma \equiv \dfrac{\gamma_1}{\gamma_3} = \dfrac{W_{L1} N_1(\overline{L}_1 + L_1^a)}{P_m X_m^a}$.

Finally, taking the natural log of sector A's gross production function and substituting in the ratio Γ, we find:[31]

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[30] Here we see that the specific division of salaried and hourly labor allows us to estimate sector A's investment parameter.

[31] Finding sector A's gross output (Q_{gross}) requires an assumption about the size of the labor adjustment costs faced by sector A. Jaramillo *et al.* (1993) find that marginal labor adjustment costs are approximately 2.3% of the annual wage per worker. Using data on the wage earned by production workers, the number of hours worked by production workers, and the total number of production worker jobs, we create the annual wage per production worker. Then, with our previously created hiring variable, we set total labor adjustment costs equal to the number of

$$2.88 \quad \ln(Q_{gross}) = \ln(\varepsilon_a) + (\Gamma\gamma_3)\ln(N_1(\bar{L}_1 + L_1^a)) + (1 - \gamma_3(1+\Gamma))\ln(K_a) + (\gamma_3)\ln(X_m^a)\,.$$

The results of the nonlinear least squares simultaneous regression, along with all of the other regression results, are presented in Table 2.4.

Next, we estimate the adjustment cost parameters (α_1 and α_2 from equation 2.36) by regressing the difference Q_{gross} less Q_{net} on net hiring squared and gross hiring squared using OLS.[32]

The final parameter to estimate is μ, the Cobb-Douglas preference parameter from the consumer's utility function. From equation 2.63, μ can be shown to be equal to the ratio of consumption expenditures over full income. The data sets used to estimate μ comes from several different sources. Data on personal consumption expenditures comes from the US Department of Commerce - Bureau of Economic Analysis - National Income and Product Accounts Tables - Table 1.1 – Gross Domestic Product. Personal consumption expenditures are converted to per capita number using a data set from the US Census Bureau. To compute full income, a data set on hourly wages from the US Department of Labor – Bureau of Labor Statistics - Current Employment Statistics is used, along

workers hired times 2.3% of the annual wage per worker. Dividing by the price of the consumer good, labor adjustment costs are converted into a quantity measure of lost output. This is then added to the observed output of sector A to create a measure of gross output (Q_{gross}).

[32] The estimation of the α_1 and α_2 parameters was excluded from the simultaneous nonlinear least squares estimation of the other production parameters in order to facilitate convergence of the nonlinear least squares regression.

with data on personal dividend income and transfer payments to persons from US Department of Commerce - Bureau of Economic Analysis, National Income and Product Accounts Tables - Table 2.8 – Personal Income by Type of Income. To create full income, hourly wages are multiplied by the full time endowment (16 hours), and then by 250 (five workdays per week, fifty workweeks per year). The result is added to the sum of per capita transfer payments and per capita dividend income. The Cobb-Douglas preference parameter, μ, is then estimated by regressing the ratio of per capita consumption expenditures to full income on a constant, using OLS.[33]

2-3-2 Steady State Computational Model

In order to implement the computational version of this model, we need to find the steady state equilibrium, which will be used for initial starting values for the intertemporal model. For this purpose, we created an Ox program to solve for the model's steady state equilibrium. We use Newton's method to solve the model, therefore the equations of the model are entered in the form LHS – RHS in order to calculate miss distances. The complete list of the steady state computational models equations is presented in Appendix 2.4.

The steady state values found by the OX program are reported in Table 2.7. These values will serve as starting point values for the intertemporal model. The next section implements and performs simulations with the intertemporal

[33] Results for the μ estimation are presented in Table 2.4.

model to see how it behaves and to gain some insight on how labor adjustment costs affect the results.

2-3-3 Intertemporal Computational Model

We implement the intertemporal model in GEMPACK. The first step of this implementation is to define the time period the model is to cover. This model covers 70 years with thirteen intervals of varying length defined by a vector of years, year(0) … year(13).[34] The unequal spacing allows us to capture the quick changes that occur immediately after the shock, without the burden of computing annual changes throughout the transition to the steady state.

With our time periods defined, we can now specify the equations of the intertemporal model. The model has three sets of equations defined over different subsets of the vector of years. All of the equations from the steady state model in Appendix 2.4, excluding the equations of motion set equal to zero, are repeated in the intertemporal model and defined over the entire time period. The equations of motion constitute the dynamic equations of the model. These equations are defined over all but the initial time period, year(0). The dynamic equations of the model are converted into their finite difference equivalents:

2.89 $$N_1(t+1) = N_1(t) + j(t) \cdot \left(H_1(t) - \delta_{n1} N_1(t)\right),$$

[34] The years used to define the intervals in this implementation of the model are, 2000, 2001, 2002, 2003, 2004, 2006, 2008, 2010, 2015, 2020, 2030, 2040, 2055, and 2070.

where j(t) is equal to year(t+1) - year(t),

$$2.90 \qquad K_a(t+1) = K_a(t) + j(t)\left(I_a(t) - \delta_{ka} K_a(t)\right)$$

$$2.91 \qquad H_1(t+1) = H_1(t) + j(t) \cdot \frac{(r+\delta_{n1})\left[\alpha_1\left(H_1(t) - \delta_{n1} N_1(t)\right) + \alpha_2 H_1(t)\right] + \frac{\overline{L}_1}{2P(t)}\left(W(t) - W_{L1}(t)\right)}{(\alpha_1 + \alpha_2)}$$

$$2.92 \qquad I_a(t+1) = I_a(t) + j(t) \cdot \frac{1}{2W_{L2}(t)\theta_a}\left[(r+\delta_k)\left[P_k(t) + 2W_{L2}(t)\theta_a I_a(t)\right] - \gamma_2\left(\varepsilon_a P(t)\right)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}(t)}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m(t)}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}\right]$$

$$2.93 \qquad K_b(t+1) = K_b(t) + j(t) \cdot \left(I_b(t) - \delta_{kb} K_b(t)\right)$$

$$2.94 \qquad I_b(t+1) = I_b(t) + j(t) \cdot \frac{(r+\delta_{kb})\left(P_k(t) + W_{L2}(t)\theta_b I_b(t)\right) - \rho(t)}{2W_{L2}(t)\theta_b},$$

The final set of equations in the intertemporal model includes the six boundary conditions, defined only for the first or last element of the vector of years. The first three boundary conditions are starting point boundary conditions comprised of the initial capital stock for sector A, $K_a(0)$, the initial number of salaried workers in sector A, $N_1(0)$, and the initial capital stock in sector B, $K_b(0)$. The other three boundary conditions require that the state variables be equal to their steady state values at the last time period. Therefore the following three equations hold only in the last period:

2.95 $$N_1^{ss}(t) = \frac{\frac{\overline{L}_1}{P(t)}\left(W(t) - W_{L1}(t)\right)}{\delta_{n1}(r + \delta_{n1})2\alpha_2}$$

2.96 $$K_a^{ss}(t) = \frac{\frac{\gamma_2}{(r + \delta_{ka})}\left(\varepsilon_a P(t)\right)^{\frac{1}{\gamma_2^a}}\left(\frac{\gamma_1^a}{W_{L1}(t)}\right)^{\frac{\gamma_1^a}{\gamma_2^a}}\left(\frac{\gamma_3^a}{P_m(t)}\right)^{\frac{\gamma_3^a}{\gamma_2^a}} - P_k(t)}{\delta_{ka}\,2W_{L2}(t)\theta_a}$$

2.97 $$K_b^{ss}(t) = \frac{1}{\delta_{kb}2W_{L2}(t)\theta_b}\left[\frac{\rho(t)}{(r + \delta_{kb})} - P_k(t)\right].$$

In order to solve the model, GEMPACK linearizes the equations of the model using Johansen's method (Harrison and Pearson, 1994). After the equations are linearized, the model is set up as a system of linear equations in the form:

2.98 $$Z\vec{x} = 0,$$

where Z is matrix with rows corresponding to the equations of the model and columns corresponding to the model's variables, and $\vec{x}$ is a vector of variables. The Z matrix is then partitioned into an endogenous and an exogenous portion:

2.99 $$Z\vec{x} = z_1\vec{x}_e + z_2\vec{x}_x = 0,$$

where z_1 is a square matrix with rows again corresponding to the equations of the model and columns corresponding to the endogenous variables and z_2 has the same number of rows and has columns corresponding to the exogenous variables of the model. The vector $\vec{x}_e$ consists of the model's endogenous variables and $\vec{x}_x$ is a vector of exogenous variables, which includes the shocked variables and thus defines the experiment. From the above equation (2.99) we know that $z_1\vec{x}_e = -z_2\vec{x}_x$. We can simply compute $-z_2\vec{x}_x$ in GEMPACK and then solve for $\vec{x}_e$ by elimination.

In order to reduce the truncation error associated with linearization, Gragg's two-step method is used for running simulations (Harrison and Pearson, 1994). Instead of simply solving the linearized model with the entire shock, Gragg's two-step method divides the shock into two equal smaller shocks. The first step considers just the first of the smaller shocks and then the derivatives are recalculated. In the second step, Gragg's method returns to the starting point and uses the derivatives calculated after the first step to find the effect of the entire shock.

2-3-4 Testing The Model

In order to test the complete model, several experiments are run to check that the model is programmed correctly. The first test is to verify that the model satisfies Walras' law. In order to perform this test, we created new variable (wal) and equated it to the left hand side minus the right hand side of the dropped market clearing condition. In both the steady state OX version of the model and the dynamic GEMPACK version, the model satisfies Walras' law. Our second test of the model is to verify that the model is homogeneous of degree 1 in prices. We ran an experiment increasing the numeraire price (P) by 50%. The effect of this experiment should be to increase all nominal variables by 50% and leave all real variable unchanged. The model does in fact produce the expected result, which indicates that the model does not include major programming errors.

2-4 RESULTS

The next step is to run simulations with the full intertemporal model. This section reports results from two different simulations. The first simulation is an announced reduction of the tax rate on implied hourly wages for salaried workers (τ_w). This simulation is comparable to the partial equilibrium phase diagram experiments in section II.A. The second simulation is an announced increase of the tax rate on intermediate goods. This simulation can be thought of as an environmental policy designed to reduce the usage of a polluting input.

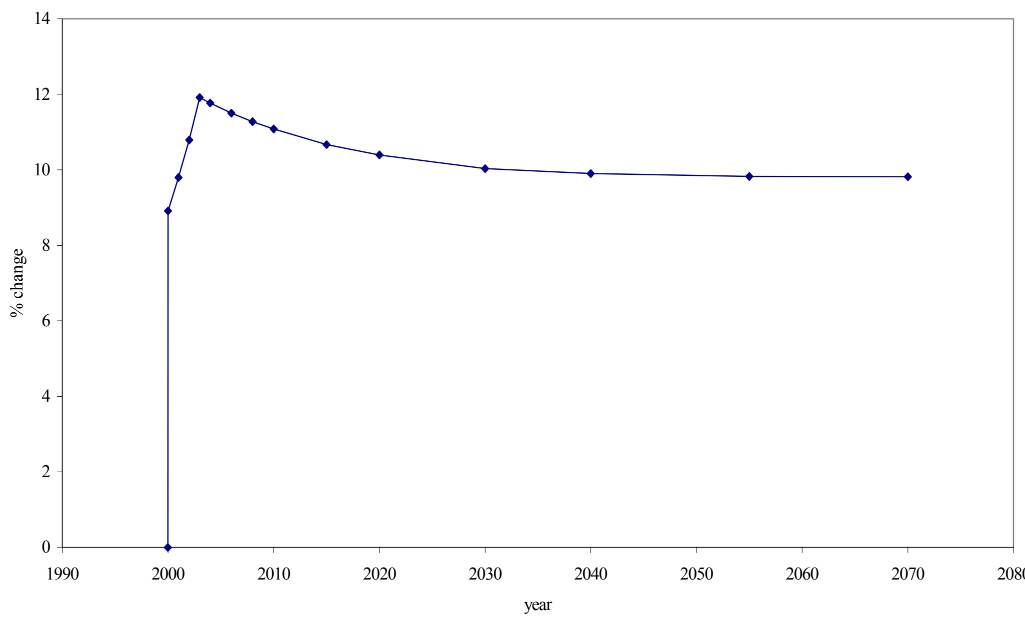
2-4-1 Reduction of t_w Simulation

The simulation results reported in this section are for an announced reduction of the τ_w, the tax rate on W, from 25.7% to 5.7%. The policy is announced in the year 2000 (period 0) and is implemented in the year 2003 (period 3). This simulation is equivalent to the partial equilibrium experiment in section 2.2.1. As with the partial equilibrium experiment, this simulation is stylized as it is primarily designed to demonstrate the behavior of this model.

Furthermore, this simulation allows for comparisons between the general equilibrium and partial equilibrium results.

The first interesting result of the intertemporal general equilibrium simulation is the time path for H_1, the hiring of salaried workers in sector A, shown in Chart 2.1.

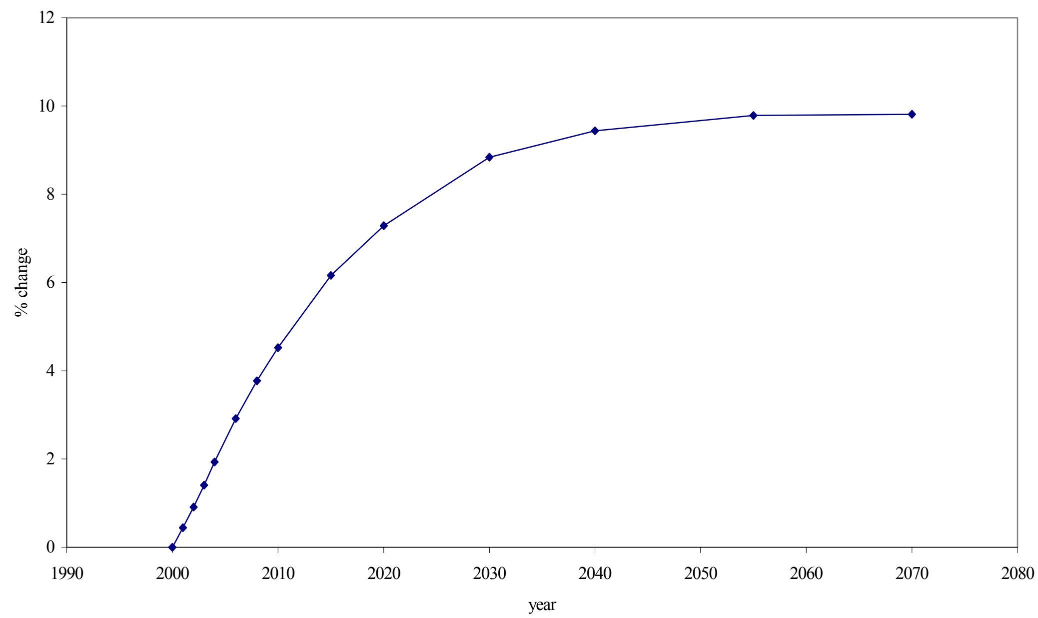
Chart 2.1 H_1 – Hiring Rate of Salaried Workers in Sector A



The first striking feature of the time path for H_1 is its difference from Figure 2.3, the integral curve for the hiring rate in the partial equilibrium experiment. In both versions of the experiment, the hiring rate experiences an initial discrete jump when the policy is announced. In the partial equilibrium experiment, the hiring rate then continues to grow as it approaches the steady state asymptotically from below, with a sharp reduction in the rate of growth when the policy is implemented. The hiring rate follows a very different trajectory in the general equilibrium version. The hiring rate of salaried workers continues to rise initially after the policy announcement, but falls when the policy is implemented. The hiring rate continues to decrease and approaches the new steady state asymptotically from above. It is immediately apparent that general equilibrium considerations have an impact on the time path of H_1.

Understanding why the time path of H_1 in this simulation is so different from the partial equilibrium version will aid the understanding of this model's behavior. The first step in tracing the source of this difference is to look at the time path of N_1, the number of salaried workers employed in sector A.

Chart 2.2 N_1 – Number of Salaried Workers in Sector A



These results for N_1 look very similar to the partial equilibrium integral curve. As with the partial equilibrium integral curve, Chart 2.2 shows N_1 increasing at a slightly increasing rate after the announcement of the new policy, and a barely perceptible inflection point in year 2004 after the new policy is implemented. After the implementation of the new policy, the number of salaried workers increases at a decreasing rate as it approaches its new steady state value.

For the firm in sector A, a reduction in τ_w has several long run implications. As a result of the new policy, the firm wants to produce more, and thus demands more inputs. Since the firm faces lower costs per salaried worker due to the decrease in τ_w, it wants to have a larger number of salaried workers, and utilize fewer flextime hours per worker. Thus, the dynamic path to the new steady state is influenced by the adjustment costs it faces when changing its stocks of capital and salaried workers.

As soon as the new policy is announced, the firm knows it would like to produce more output. Since it is unable immediately to increase the amount of capital or the number of workers it uses, the first response is to increase its use of intermediate goods.

Chart 2.3 X_m^a – Intermediate Goods Used in Sector A

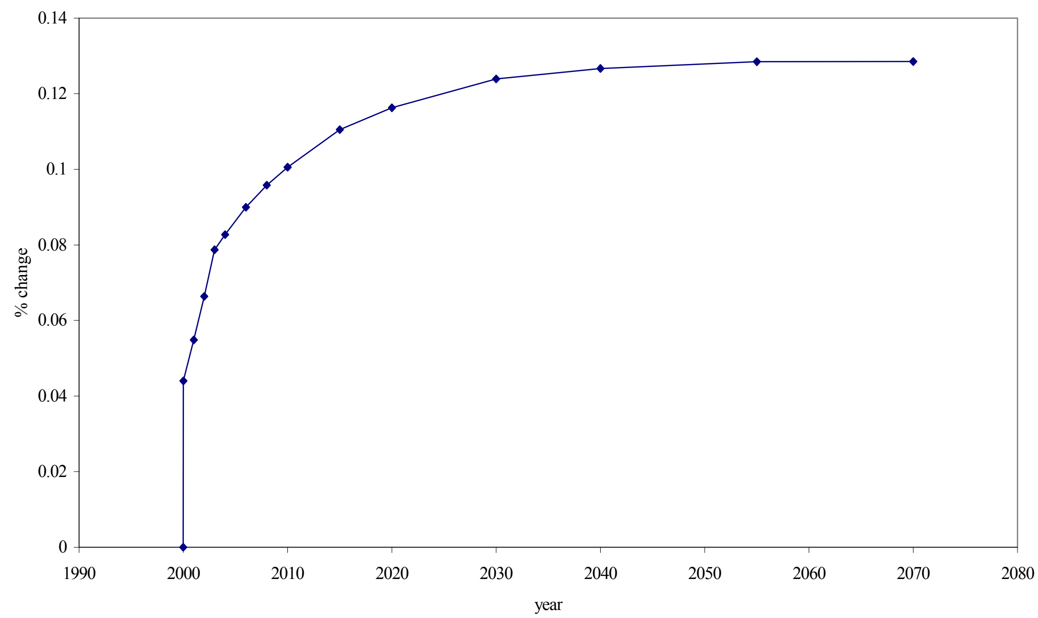
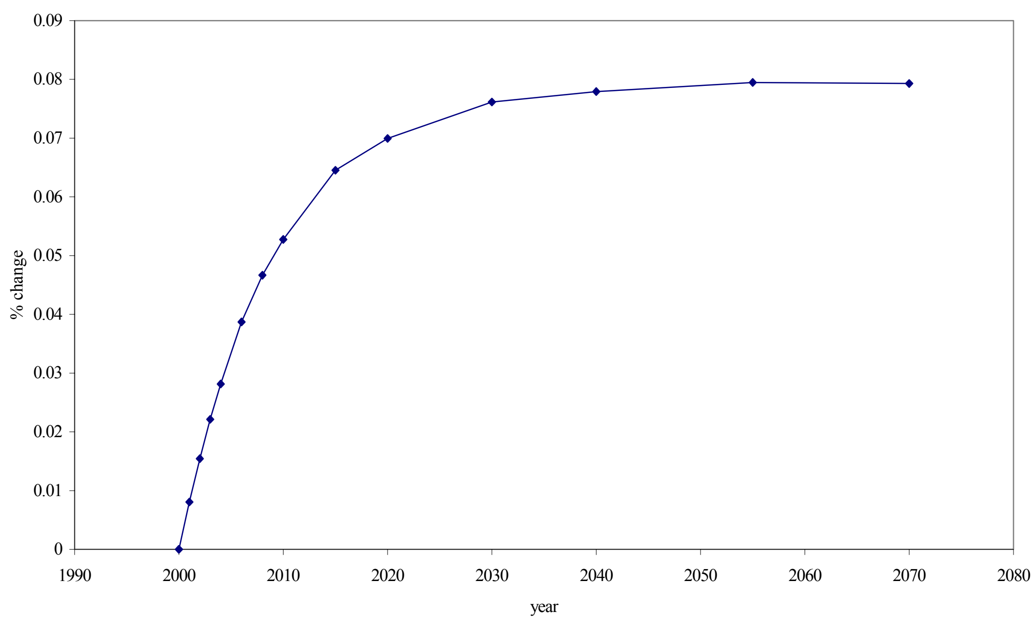


Chart 2.3 shows the firm's initial increase in intermediate goods used in production, and its path to the new steady state using slightly more intermediate goods. In

Chart 2.4 we see the firms increased use of capital over time.

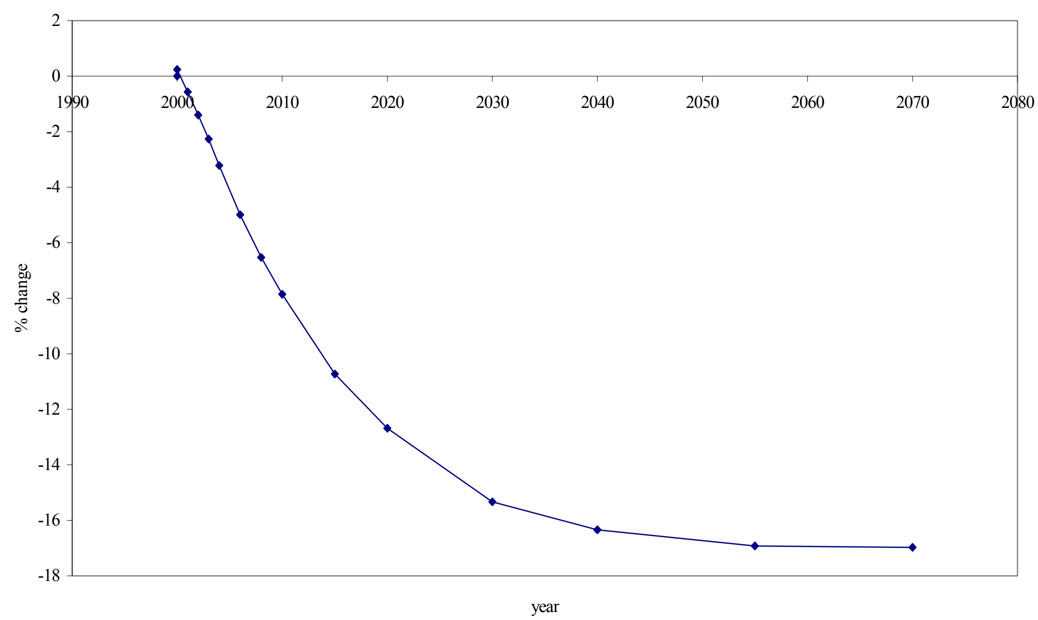
Chart 2.4 K_a – Sector A Capital Used in Production



To complete the picture of the firm's inputs,

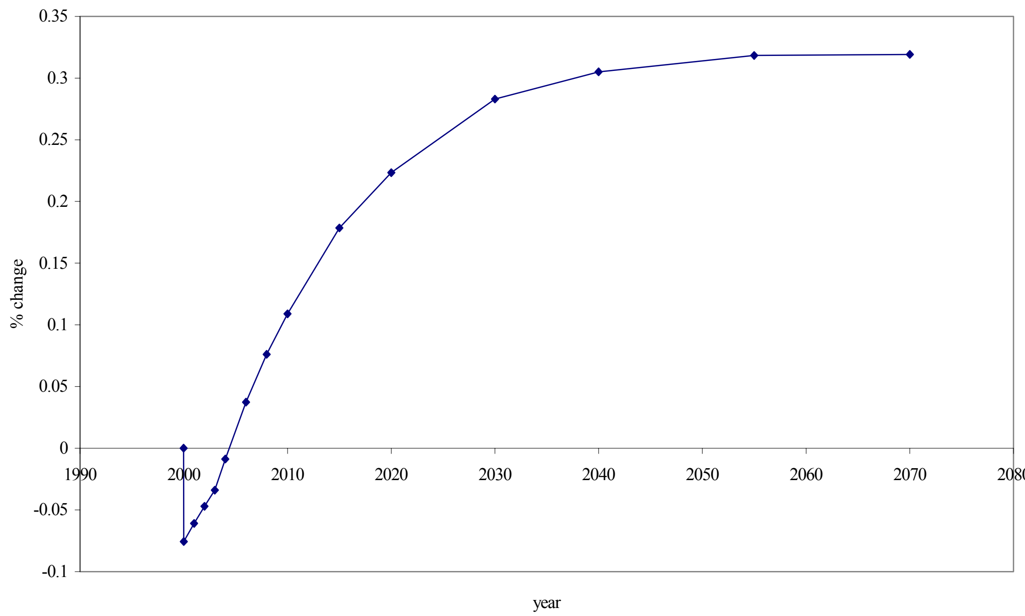
Chart 2.5 shows how the firm's use of flextime hours per worker changes as more workers are hired. At first there is a slight increase in flextime labor, since the firm cannot immediately increase the number of workers. After the initial small increase, which may or may not be significant, the use of flextime labor declines to its new steady state value.

Chart 2.5 L_1^a – Flextime Hours Used in Sector A per Salaried Worker



The dynamic path of the firm's output is shown in Chart 2.6. The initial fall in output is due to the adjustment costs incurred by the firm as it increases its number of salaried employees. As the firm goes through the adjustment process, the output increases to its new steady state level.

Chart 2.6 Q_{net} – Sector A Net Output of Consumer Goods



One of the keys to understanding why time paths of the control and state variables appear so different in the partial and general equilibrium versions of this experiment is the way the firm's decreased use of flextime hours (L_1^a) affects the flextime wage rate (W_{L1}).

Chart 2.7 W_{L1} – Salaried Worker Flextime Wage Rate

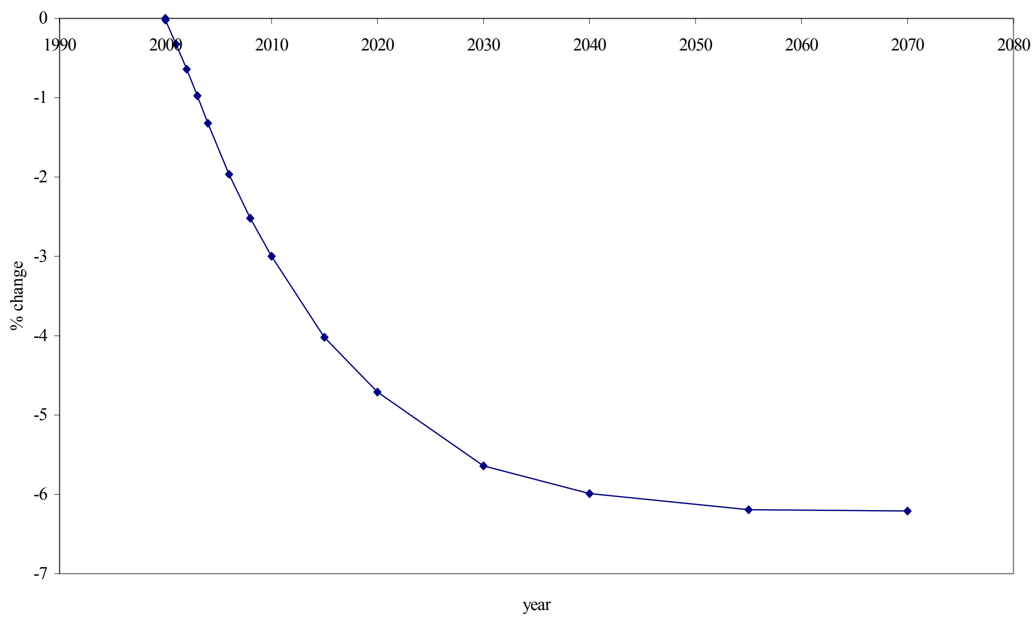


Chart 2.7 shows that as the firm hires more workers and its demand for flextime hours falls, the equilibrium flextime wage rate falls as well.

The final source of the difference between the dynamic paths from the partial equilibrium experiment and the general equilibrium simulation is the implied hourly wage rate for salaried workers (W), which was subjected to the tax decrease.

Chart 2.8 W – Salaried Worker Implied Hourly Wage Rate

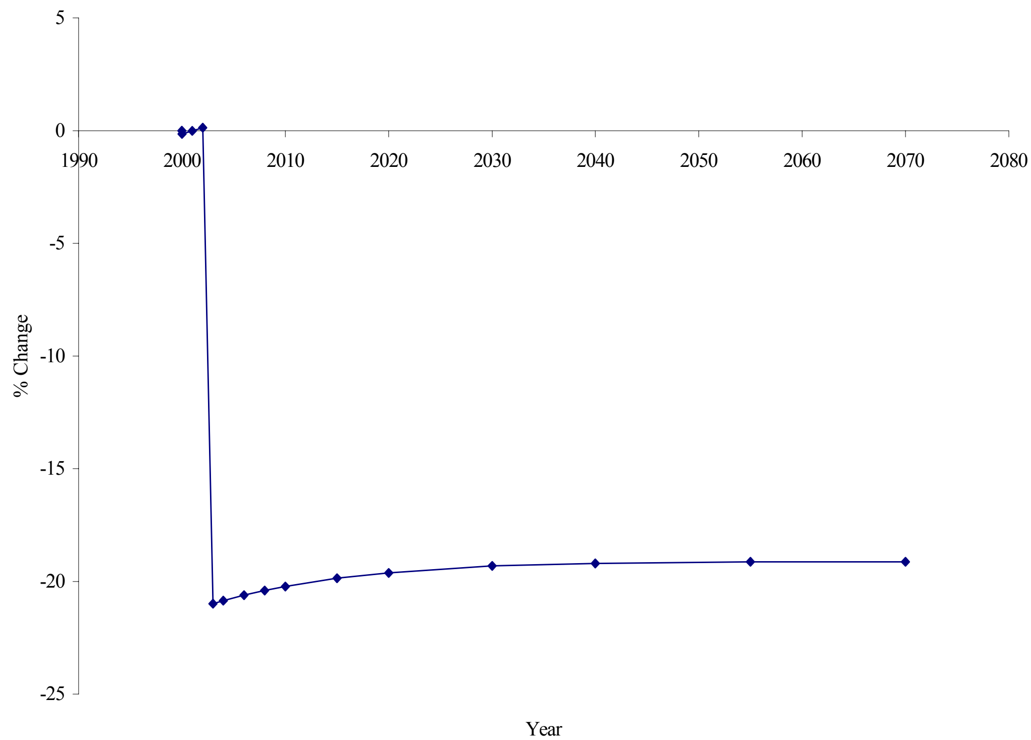


Chart 2.8 shows that W drops quite dramatically when the policy is implemented and then rebounds. The implied hourly wage rate for salaried workers paid by sector A is expected to fall when τ_w is decreased. The other rebound visible in Chart 2.8 is due to the efficiency wage condition. As Chart 2.2 shows, the number of salaried workers is increasing, so conversely the number of hourly workers is decreasing. With fewer hourly workers, the wage rate for hourly

workers increases. As the wage rate for hourly workers increases, salaried workers need to be paid more in order to ensure they choose not to shirk.

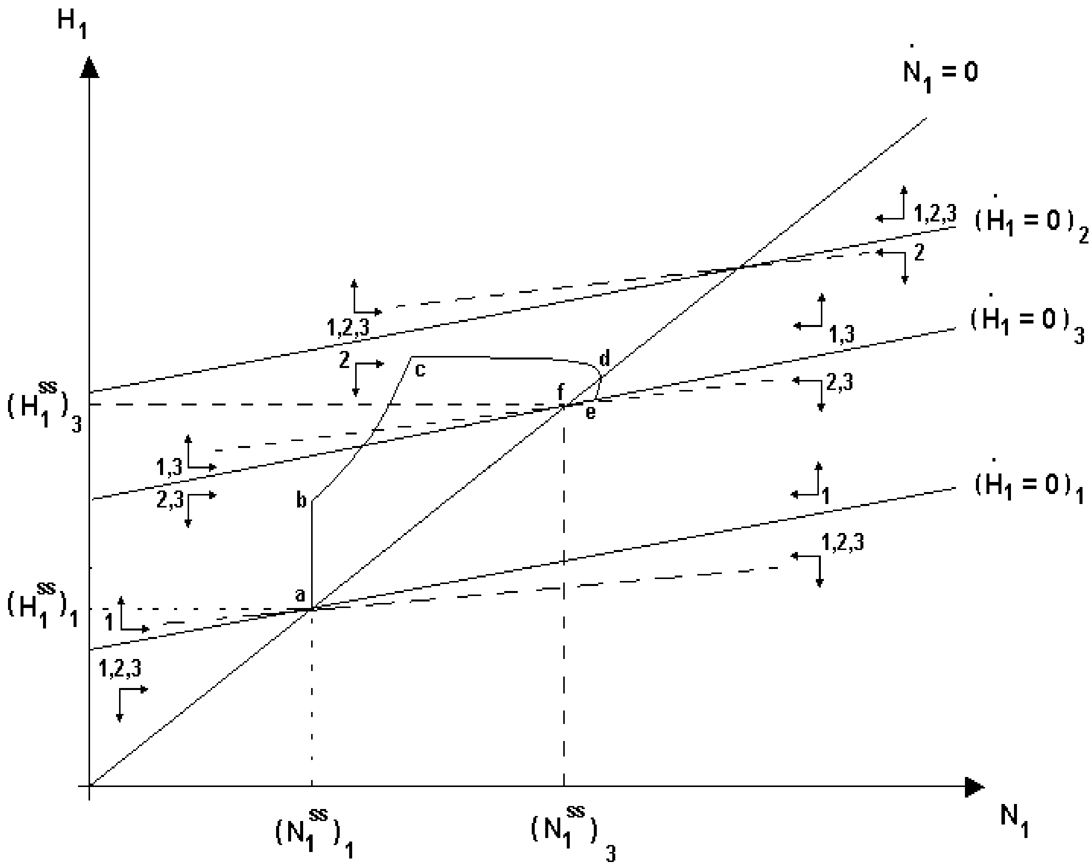
The dynamic paths of W and W_{L1} have important implications for the dynamic paths of H_1 and N_1. Equation 2.116 from Appendix 2.2, reproduced below as equation 2.100, describes the $\dot{H}_1 = 0$ isocline.

$$2.100 \quad \dot{H}_1 = 0 \Rightarrow H_1 = \frac{\frac{\overline{L}_1}{P}(W_{L1} - W)}{2(\alpha_2 + \alpha_3)} + \frac{\alpha_2 \delta_{n1}}{(\alpha_2 + \alpha_3)} N_1$$

In the partial equilibrium analysis, W_{L1} was assumed to be constant, whereas Chart 2.7 shows that it is clearly not constant in the general equilibrium simulation. Furthermore, the partial equilibrium analysis assumed W decreased in a single step. The general equilibrium simulation shows that W rebounds after the initial decrease. The implication is that as time passes, since W_{L1} is decreasing and W rebounds, the H_1 intercept of the $\dot{H}_1 = 0$ isocline is decreasing. The phase diagram from the partial equilibrium experiment does not capture everything that was actually happening. To describe fully what happens as a result of an announced reduction of τ_w, a phase diagram experiment needs to include in addition to an announced reduction of τ_w, a series of small anticipated negative shocks to W_{L1}, and a later small anticipated positive shock to W. Figure 2.6 shows an approximation of this result with fall of W_{L1} and rebound of W occurring in a single anticipated shock instead of in a series of small shocks.[35]

[35] This example is consistent with the model's assumptions about firms having perfect foresight.

Figure 2.6 Revised Effects of an Announced Decrease of τ_w

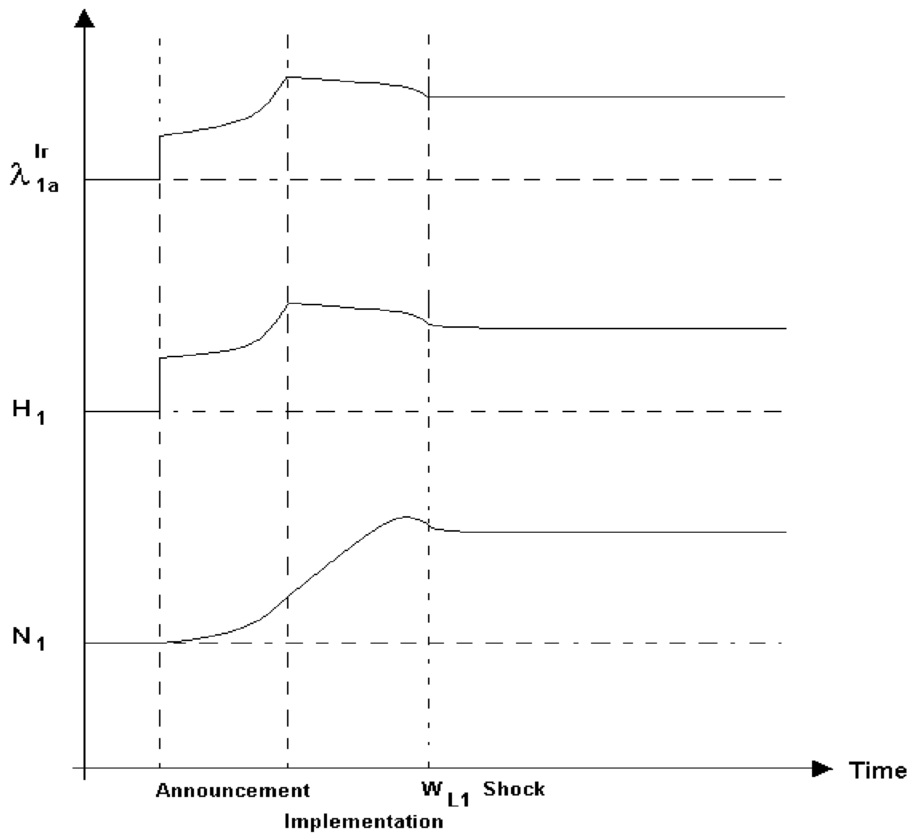


In this diagram, the firm begins at point a, the initial steady state. When the new policy is announced, the firm immediately increases its hiring rate and jumps from point a to point b.[36] In the period between the announcement of the implementation of the policy, the firm's hiring rate and stock of salaried

[36] The firm's initial increase in its hiring rate accounts for its anticipation of the new policy and its anticipation of the subsequent shocks to W_{L1} and W. Since the firm has perfect foresight and anticipates the subsequent shocks, it will evolve towards the eventual steady sate without any further discrete jumps in its hiring rate.

employees follows the path from point b to point c as governed by the $(\dot{H}_1 = 0)_1$ isocline. When the policy is implemented, the firm begins to evolve from point c through point d to point e, along a curve governed by the $(\dot{H}_1 = 0)_2$ isocline. At point d, the curve crosses the $\dot{N}_1 = 0$ isocline, and the number of workers begins to decrease. Then at point e, the firm is hit by the anticipated shocks reducing the flextime wage rate (W_{L1}) and a slight rebound of the implied wage rate for salaried workers (W). At this point the firm finds itself on the saddle path governed by the $(\dot{H}_1 = 0)_3$ isocline, evolving towards the new steady state. Figure 2.7 shows the integral curves for this experiment.

Figure 2.7 Integral Curves for Revised τ_w Experiment



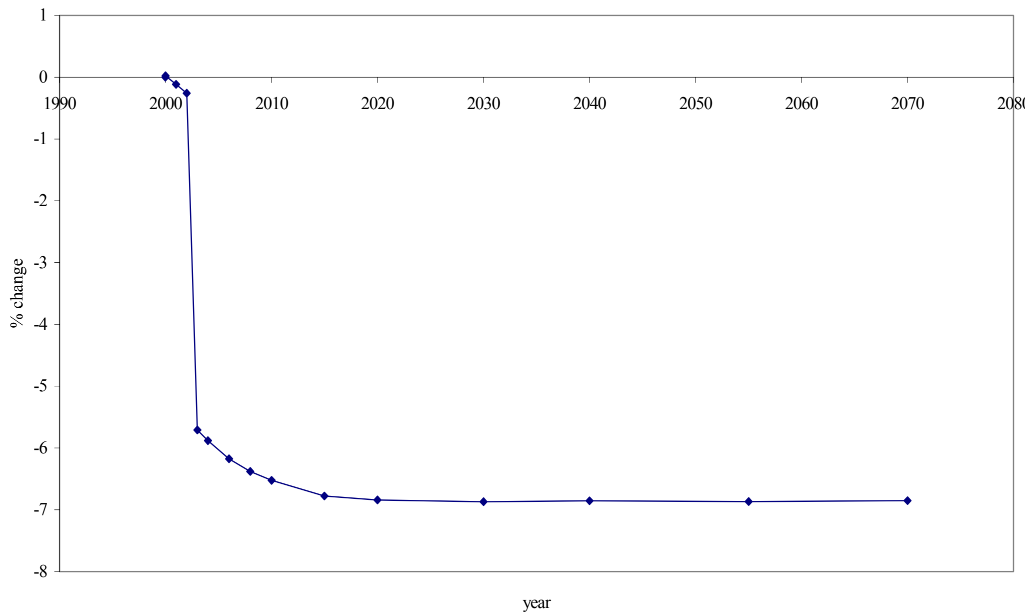
These integral curves show agreement with Chart 2.1 and Chart 2.2. As with the computational experiment, the integral curve for H_1 approaches the steady state value from above. Furthermore, in both versions, H_1 peaks when the policy is implemented and N_1 reaches a peak several years later. Extending this type of analysis to include a series of small shocks to W_{L1} and W instead of a single shock, we would expect the integral curves to appear much like the results from the computational experiment.

2-4-2 Increase of the Tax Rate on Intermediate Goods (τ_{xm}) Simulation

The second simulation in this chapter corresponds to a policy designed to discourage the use of intermediate goods. Such a policy may be desirable if production or use of the intermediate good is associated with a negative environmental externality. This experiment can be thought of as an energy tax. The specific simulation is an increase of the tax rate on intermediate goods (τ_{xm}) from 5% to 15%.[37] The policy is announced in the year 2000 (period 0) and is implemented in the year 2003 (period 3). A few of the interesting results from this simulation are presented in this section. First, Chart 2.9 shows how the announced increase of the tax rate on intermediate goods affects the usage of intermediate goods in sector A. The use of intermediate goods remains stable after the tax increase is announced, and then falls when the tax increase goes into effect.

[37] These numbers are not associated with an actual policy, as existing taxes on intermediate goods are very low. The simulation is simply intended to demonstrate the behavior of the model.

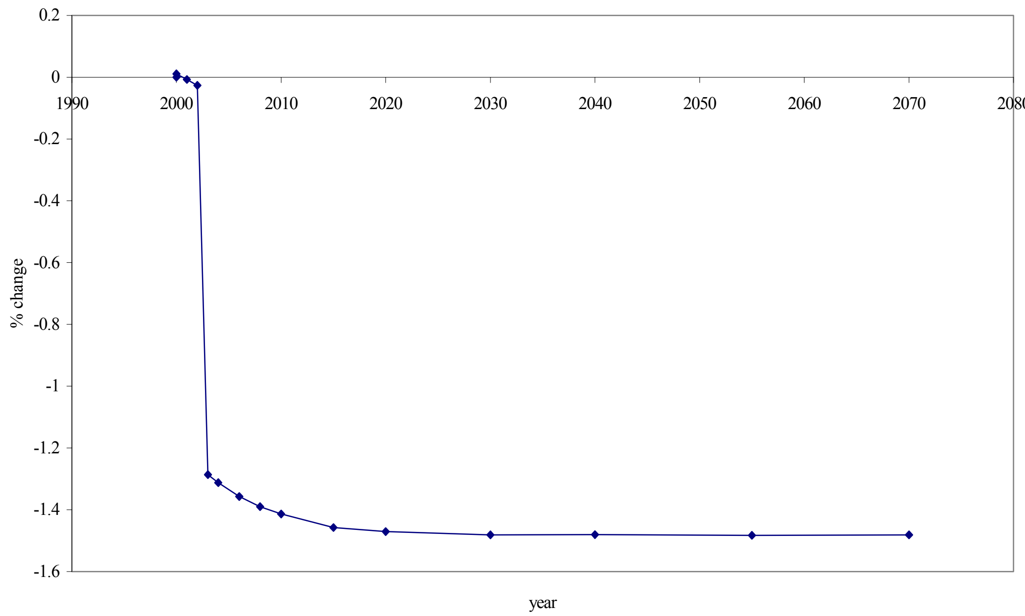
Chart 2.9 X_m^a – Intermediate Goods Used in Sector A



Due to the increased tax rate on intermediate goods, costs for sector A have increased, so the price increases, and the demand decreases. Thus, sector A produces less using relatively fewer intermediate goods and relatively more labor and capital.

Chart 2.10 shows the reduction in output due to the increase of the tax rate on intermediate goods (τ_{xm}).

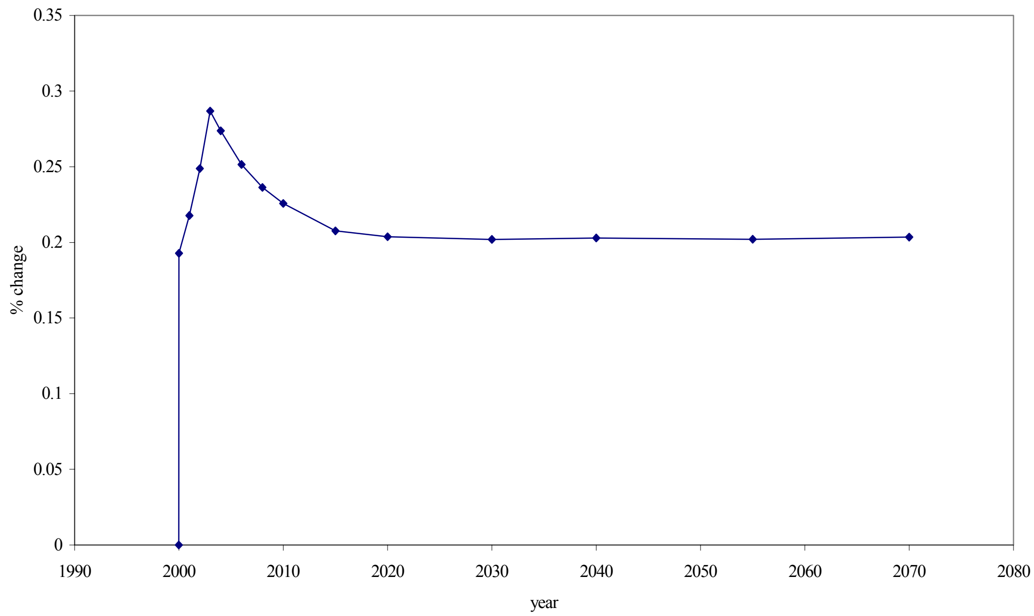
Chart 2.10 Q_{net} – Output of Sector A



Output remains largely unchanged when the policy is announced, and then falls dramatically when the tax increase goes into effect.

Chart 2.11 shows how investment in sector A is affected by the new policy.

Chart 2.11 I_a – Investment in Sector A



Investment in sector A increases immediately when the new policy is announced. After quickly reaching a high level, investment falls back to the level needed to sustain the new larger capital stock.

The evolution of the capital stock is shown in Chart 2.12.

Chart 2.12 K_a – Capital Stock in Sector A

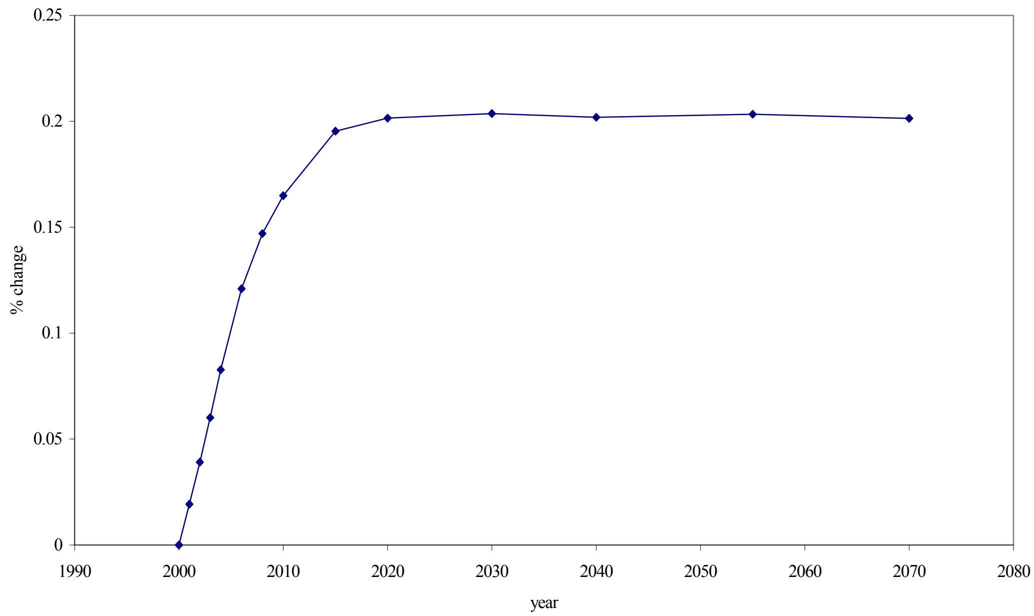
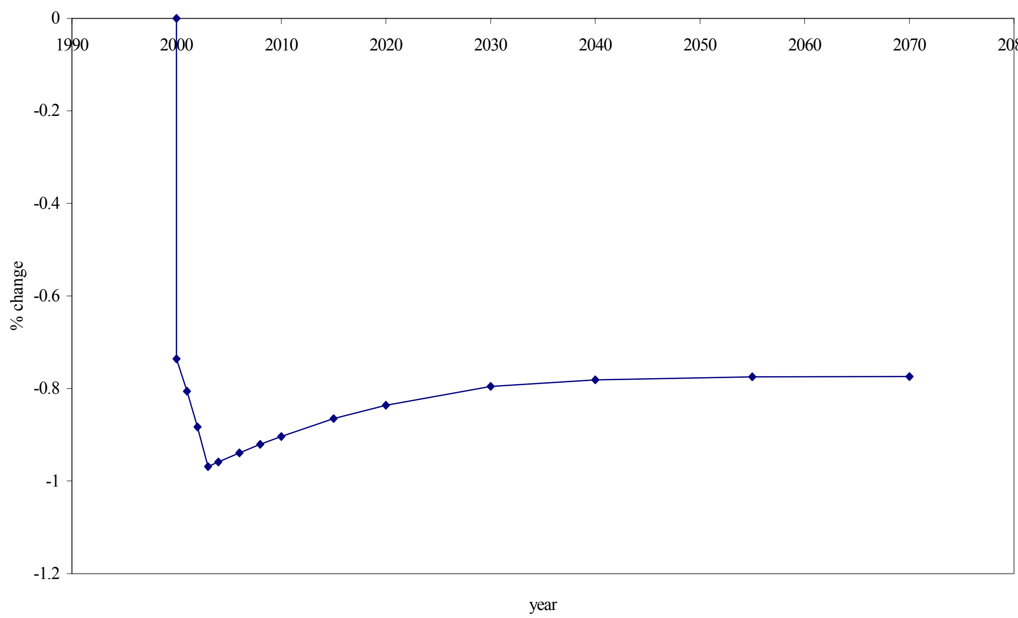


Chart 2.13 now shows how the hiring rate of salaried workers in sector A is affected by the new policy.

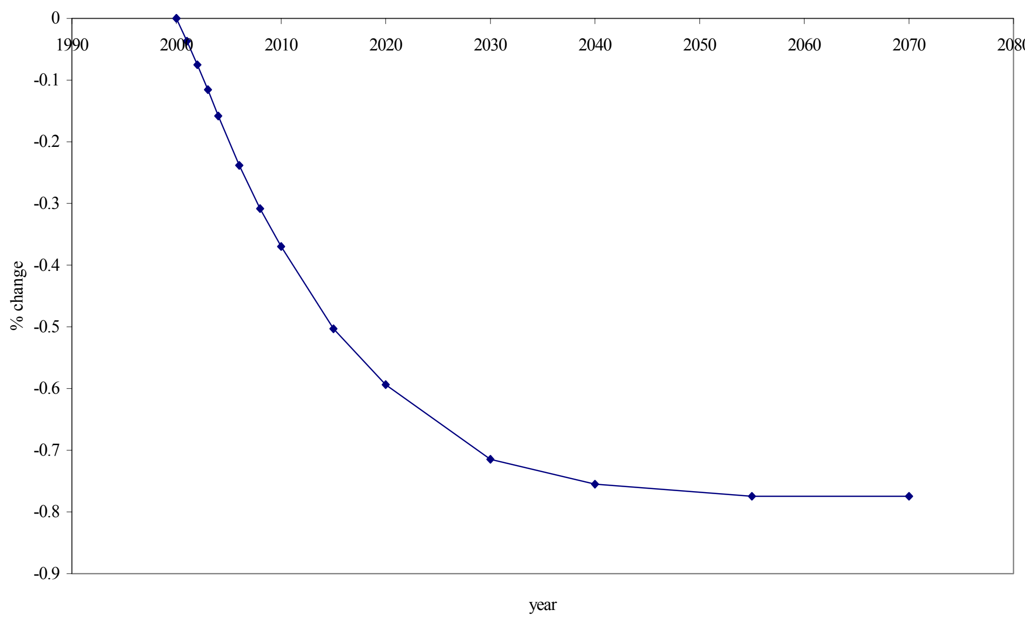
Chart 2.13 H_1 – Hiring Rate of Salaried Workers in Sector A



The hiring rate of salaried workers falls when the new policy is announced, and then falls further until the policy takes effect. Once the policy is in effect, the hiring rate falls slightly before rebounding towards the new steady state level.

Finally the actual number of salaried workers in sector A is shown in Chart 2.14.

Chart 2.14 N_1 – Number of Salaried Workers in Sector A



The number of salaried employees falls to the new steady state value. While the number of salaried employees decreases, due to the reduced output, the reduction is smaller than that for intermediate goods.

The results presented here are a small sample of the output of this simulation. The results give a feel for how a shock to one component of the system can have repercussions throughout the system. Furthermore, the time paths of the variables are shown explicitly. This allows one to examine the full intertemporal impact of the policy.

In order to judge the desirability of a policy, or compare two different polices, we need to calculate the equivalent variation, which is simply the amount of additional expenditures needed to achieve the new utility level at the old prices. For this model, we need to consider how the policy affects the entire time path of future expenditures. The expression for the intertemporal equivalent variation is:

$$
\begin{aligned}
2.101 \quad EV = \int_0^{\infty} [& N_{11}(t)U_1(t)\left(\frac{P(1+\tau_q)}{\mu}\right)^{\mu}\left(\frac{W_{L1}(0)(1-\tau_{L1})}{1-\mu}\right)^{1-\mu} \\
& + N_{21}(t)U_1(t)\left(\frac{P(1+\tau_q)}{\mu}\right)^{\mu}\left(\frac{W_{L2}(0)(1-\tau_{L2})}{1-\mu}\right)^{1-\mu} \\
& + N_{12}(t)U_2(t)\left(\frac{P(1+\tau_q)}{\mu}\right)^{\mu}\left(\frac{W_{L1}(0)(1-\tau_{L2})}{1-\mu}\right)^{1-\mu} \\
& + N_{22}(t)U_2(t)\left(\frac{P(1+\tau_q)}{\mu}\right)^{\mu}\left(\frac{W_{L2}(0)(1-\tau_{L2})}{1-\mu}\right)^{1-\mu} \\
& - N_1(0)U_1(0)\left(\frac{P(1+\tau_q)}{\mu}\right)^{\mu}\left(\frac{W_{L1}(0)(1-\tau_{L1})}{1-\mu}\right)^{1-\mu} \\
& - N_2(0)U_2(0)\left(\frac{P(1+\tau_q)}{\mu}\right)^{\mu}\left(\frac{W_{L2}(0)(1-\tau_{L2})}{1-\mu}\right)^{1-\mu}]e^{-rt}dt \ ,
\end{aligned}
$$

where $N_{11}(t)$ is the number of salaried workers in period t who were also salaried workers in period 0, $N_{21}(t)$ is the number of salaried workers in period t who were hourly workers in period 0, $N_{12}(t)$ is the number of hourly workers in period t who were salaried workers in period 0, and $N_{22}(t)$ is the number of hourly workers in period t who were also hourly workers in period 0. This accounting is needed to keep track of each worker's prices in the initial period, and utility in the current period. Using the discrete data from the simulation, we can approximate the equivalent variation resulting from the increase of the tax on intermediate goods (τ_{xm}).

Table 2.1 Equivalent Variation

EV	G Old	G New	G Change	MCPF
-1.92E+07	1.95E+09	1.98E+09	3.50E+07	1.55

The equivalent variation resulting from the increase of the tax rate on intermediate goods (τ_{xm}) is approximately negative $19 million. To put this number into context, the present value of the total future government expenditures in this model are on the order of $1.9 billion, and the present value of the additional revenue generated by the tax increase is approximately $35 million.[38] This implies that for this tax increase, every dollar of additional revenue generated costs approximately $1.55.[39]

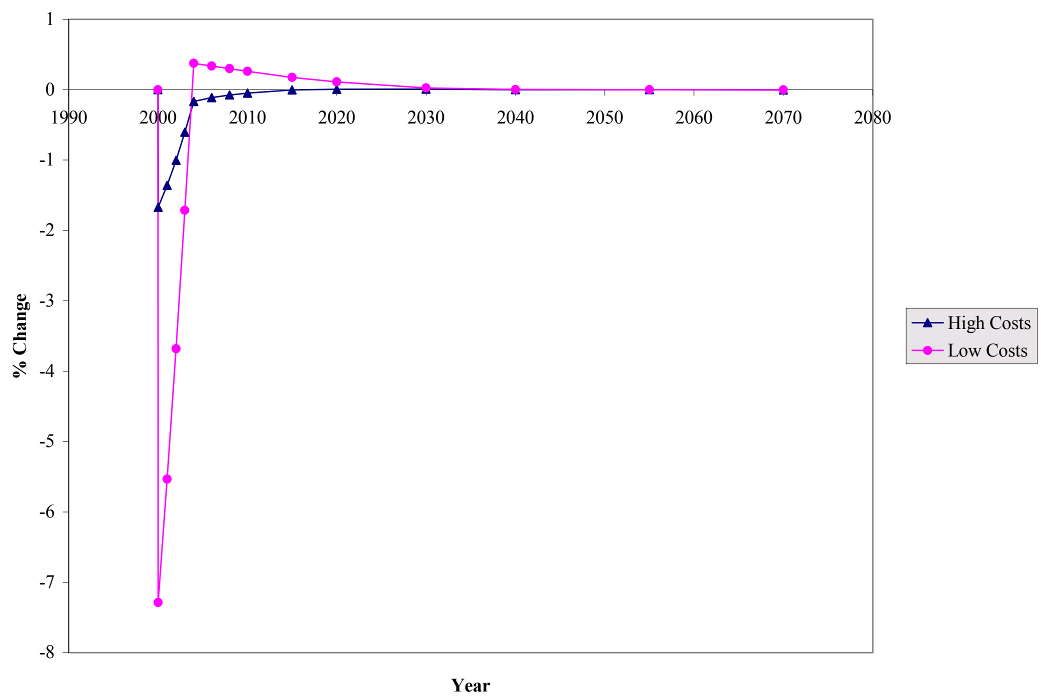
2-4-3 Temporary Increase of the Tax on Consumer Goods (τ_s) Simulation

The last simulation differs from the first two in that it does not use the estimated values for the labor adjustment cost parameters (α_1 and α_2). Instead this simulation is run twice, once with low values for the labor adjustment cost

[38] The value for total government expenditures is small due to the incomplete tax structure.
[39] For a small increase in the tax rate this would be the marginal cost of public funds (MCPF).

parameters, and once with high values for the labor adjustment cost parameters.[40] This allows us to see the impact of labor adjustment costs. The policy simulated here is an immediate temporary increase of the tax on consumer goods (τ_s) from 5% to 25%. The policy goes into effect in the year 2000 as soon as it is announced, and lasts until 2003. In 2004 the tax rate on consumer goods returns to its initial level. The effects of this policy on the firm's hiring of workers into salaried jobs is shown in Chart 2.15:

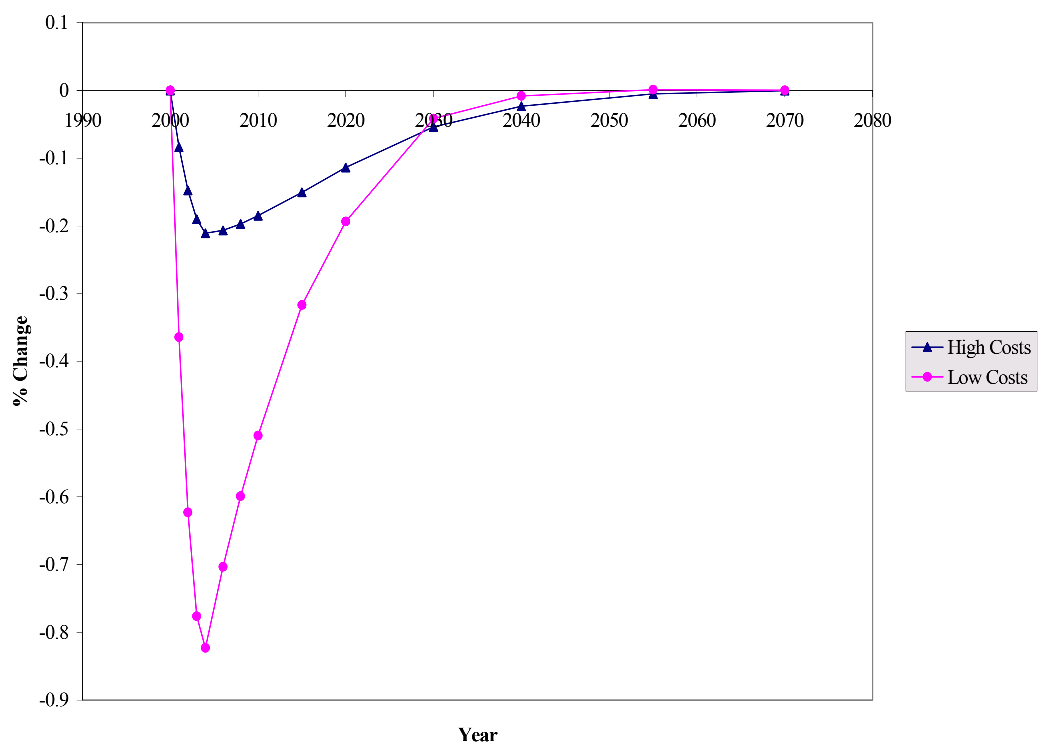
Chart 2.15 H_1 – Hiring Rate of Salaried Workers – High & Low Labor Adjustment Costs



[40] The low labor adjustment cost simulation sets $\alpha_1 = \alpha_2 = .005$, and the high labor adjustment cost simulation sets $\alpha_1 = \alpha_2 = .233$.

Here we see that the hiring rate of workers into salaried jobs changes much more dramatically when adjustment costs are low than when they are high. When adjustment costs are low, the firm reduces the number of salaried jobs more quickly when the policy is announced, and hires workers back into salaried jobs more quickly once the tax returns to normal. C shows the effects of the policy on the number of salaried jobs:

Chart 2.16 N_1 – Number of Salaried Jobs – High & Low Labor Adjustment Costs



This chart dramatically demonstrates the importance of labor adjustment costs. Even a temporary policy or shock that effects a particular industry can have long lasting effects on the number of workers in that industry. Capturing the magnitude and duration of these effects requires that we include labor adjustment costs in our models. This is particularly important when modeling environmental policies that strongly impact a narrow portion of the economy. These policies have the potential to strongly impact the number of jobs in the affected industries. Accurately describing the desirability of such a policy requires a model that has the ability to capture both the general equilibrium effects of the policy, and the transitional dynamics driven by labor adjustment costs.

2-5 CONCLUSION

This chapter has introduced a method for including labor adjustment costs in an intertemporal computable general equilibrium model. The simulations presented in this chapter provide a feel for how this model behaves. They demonstrate the importance of considering labor adjustment costs when considering potential policy proposals. Furthermore, the simulations presented in this chapter clearly show the importance of considering labor adjustment costs in a general equilibrium setting. Policy makers need to consider not only the long-term effects of their proposals, but also the transitional effects. A model such as this one is needed to take these effects into account. The inclusion of adjustment costs and the calculation of equivalent variations for the entire time path, allows an accurate picture of the true cost of a policy to be obtained.

The possibilities for extending this research are numerous. One important extension is to include labor adjustment costs in all sectors. This would allow the model to capture labor adjustment costs involved with workers changing types of jobs, or adjustment costs associated with workers staying in the same type of job and changing industries. Another extension, including labor adjustment costs for the labor used to install capital, could reveal some important interactions between the capital and labor adjustment costs. It is also possible to add an education decision to the model. This type of model could potentially have populations of people with different abilities who choose what type of job to train for with their education decision. Different job categories could then be added to the model. With an education decision in the model, it is possible that workers could bear some of the adjustment costs involved with change careers. It may also be possible to include unemployment in this type of model. All of these features could also be added to a larger scale computable general equilibrium model of the U.S. economy. The labor adjustment costs introduced into an intertemporal computable general equilibrium model have great potential for enriching CGE models by allowing them to accurately capture transition costs.

Tables

Table 2.2 192-Order Industries and Mapping to 35-Order Industries

192-0RDER INDUSTRY NUMBER	DESCRIPTION	35-ORDER INDUSTRY NUMBER	STANDARD INDUSTRIAL CALSSIFICATION (SIC) 1987
	Agriculture, forestry, fisheries		
1	Agricultural production	1	01,02
2	Veterinary services	1	74
3	Landscape and horticultural services	1	78
4	Agric., forestry and fisheries services, nec.	1	07 (excl. 074&078)
5	Forestry, fishing, hunting, & trapping	1	08,09
	Mining		
6	Metal mining	2	10
7	Coal mining	3	12
8	Crude petroleum, natural gas, and gas liquids	4	131,132
9	Oil and gas field services	4	138
10	Nonmetallic minerals, except fuels	5	14
	Construction		
11	Construction	6	15,16,17
	Manufacturing		
12	Logging	11	241
13	Sawmills and planing mills	11	242
14	Millwork, plywood, and structural members	11	243
15	Wood containers and misc. wood products	11	244,249
16	Wood buildings and mobile homes	11	245
17	Household furniture	12	251

18	Partitions and fixtures	12	254
19	Office and misc. furniture and fixtures	12	252,253,259
20	Glass and glass products	19	321,322,323
21	Hydraulic cement	19	324
22	Stone, clay, and misc. mineral products	19	325,326,328,329
23	Concrete, gypsum, & plaster products	19	327
24	Blast furnaces and basic steel products	20	331
25	Iron and steel foundries	20	332
26	Primary nonferrous smelting & refining	20	333
27	All other primary metals	20	334,339
28	Nonferrous rolling and drawing	20	335
29	Nonferrous foundries	20	336
30	Metal cans and shipping containers	21	341
31	Cutlery, hand tools, and hardware	21	342
32	Plumbing and nonelectric heating equipment	21	343
33	Fabricated structural metal products	21	344
34	Screw machine products, bolts, rivets, etc.	21	345
35	Metal forgings and stampings	21	346
36	Metal coating, engraving, and allied services	21	347
37	Ordnance and ammunition	25	348
38	Miscellaneous fabricated metal products	21	349
39	Engines and turbines	22	351
40	Farm and garden machinery and equipment	22	352
41	Construction and related machinery	22	353
42	Metalworking machinery and equipment	22	354
43	Special industry machinery	22	355
44	General industrial machinery and equipment	22	356
45	Computer and office equipment	23	357
46	Refrigeration and service industry machinery	23	358
47	Industrial machinery, nec.	22	359
48	Electric distribution equipment	23	361
49	Electrical industrial apparatus	23	362
50	Household appliances	23	363
51	Electric lighting and wiring equipment	23	364
52	Household audio and video equipment	23	365
53	Communications equipment	23	366
54	Electronic components and accessories	23	367
55	Miscellaneous electrical equipment	23	369

56	Motor vehicles and equipment	24	371
57	Aerospace	23	372,376
58	Ship and boat building and repairing	25	373
59	Railroad equipment	25	374
60	Miscellaneous transportation equipment	25	375,379
61	Search and navigation equipment	26	381
62	Measuring and controlling devices	26	382
63	Medical equip., instruments, and supplies	26	384
64	Ophthalmic goods	26	385
65	Photographic equipment and supplies	26	386
66	Watches, clocks, and parts	26	387
67	Jewelry, silverware, and plated ware	27	391
68	Toys and sporting goods	27	394
69	Manufactured products, nec.	27	393,395,396,399
70	Meat products	7	201
71	Dairy products	7	202
72	Preserved fruits and vegetables	7	203
73	Grain mill products and fats and oils	7	204,207
74	Bakery products	7	205
75	Sugar and confectionery products	7	206
76	Beverages	7	208
77	Miscellaneous food and kindred products	7	209
78	Tobacco products	8	21
79	Weaving, finishing, yarn, and thread mills	9	221-224,226,228
80	Knitting mills	9	225
81	Carpets and rugs	9	227
82	Miscellaneous textile goods	9	229
83	Apparel	10	231-238
84	Miscellaneous fabricated textile products	10	239
85	Pulp, paper, and paperboard mills	13	261-263
86	Paperboard containers and boxes	13	265
87	Converted paper products except containers	13	267
88	Newspapers	14	271
89	Periodicals	14	272
90	Books	14	273
91	Miscellaneous publishing	14	274
92	Commercial printing and business forms	14	275,276
93	Greeting cards	14	277

94	Blankbooks and bookbinding	14	278
95	Service industries for the printing trade	14	279
96	Industrial chemicals	15	281,286
97	Plastics materials and synthetics	17	282
98	Drugs	15	283
99	Soap, cleaners, and toilet goods	15	284
100	Paints and allied products	15	285
101	Agricultural chemicals	15	287
102	Miscellaneous chemical products	15	289
103	Petroleum refining	16	291
104	Miscellaneous petroleum and coal products	16	295,299
105	Tires and inner tubes	17	301
106	Rubber products, plastic hose, and footwear	17	302,305,306
107	Miscellaneous plastics products, nec.	17	308
108	Footwear, except rubber and plastic	18	313,314
109	Luggage, handbags, leather products, nec.	18	311,315-317,319
	Transportation		
110	Railroad transportation	28	40
111	Local and interurban passenger transit	28	41
112	Trucking and courier services except air	28	421,423
113	Warehousing and storage	28	422
114	Water transportation	28	44
115	Air transportation	28	45
116	Pipelines, except natural gas	28	46
117	Passenger transportation arrangement	28	472
118	Miscellaneous transportation services	28	473,474,478
	Communications		
119	Telephone & telegraph communications and communications services, nec.	29	481-2,489
120	Cable and pay television services	29	484
121	Radio and TV Broadcasting	29	483
	Utilities		

122	Electric utilities	30	491
123	Gas utilities	31	492
124	Combined Utilities	30	493
125	Water and sanitation	31	494-497
	Trade		
126	Wholesale trade	32	50,51
127	Retail trade exc. eating and drinking places	32	52-57,59
128	Eating and drinking places	32	58
	Finance, Insurance, and Real Estate		
129	Depository institutions	33	60
130	Nondepository; holding & investment offices	33	61,67
131	Security and commodity brokers	33	62
132	Insurance carriers	33	63
133	Insurance agents, brokers, and service	33	64
134	Real estate	33	65
135	Royalties	33	n.a.
136	Owner-occupied dwellings	33	n.a.
	Services		
137	Hotels	34	701
138	Other lodging places	34	702-704
139	Laundry, cleaning, and shoe repair	34	721,725
140	Personal services, nec.	34	722,729
141	Beauty and barber shops	34	723,724
142	Funeral service and crematories	34	726
143	Advertising	34	731
144	Services to buildings	34	734
145	Miscellaneous equipment rental and leasing	34	735
146	Personnel supply services	34	736
147	Computer and data processing services	34	737
148	Miscellaneous business services	34	732,733,738
149	Automotive rentals, without drivers	34	751
150	Automobile parking, repair, and services	34	752-754

151	Electrical repair shops	34	762
152	Watch, jewelry, & furniture repair	34	763,764
153	Miscellaneous repair services	34	769
154	Motion pictures	34	781-783
155	Video tape rental	34	784
156	Producers, orchestras, and entertainers	34	792
157	Bowling centers	34	793
158	Commercial sports	34	794
159	Amusement and recreation services, nec.	34	791,799
160	Offices of health practitioners	34	801-804
161	Nursing and personal care facilities	34	805
162	Hospitals	34	806
163	Health services, nec.	34	807-809
164	Legal services	34	81
165	Educational services	34	82
166	Individual and miscellaneous social services	34	832,839
167	Job training and related services	34	833
168	Child day care services	34	835
169	Residential care	34	836
170	Museums, botanical, zoological gardens	34	84
171	Membership organizations	34	86
172	Engineering and architectural services	34	871
173	Research and testing services	34	873
174	Management and public relations	34	874
175	Accounting, auditing, and other services	34	872,89
176	Private households	34	88
	Government		
177	U.S. Postal Service	35	NA
178	Federal electric utilities	35	NA
179	Federal government enterprises, nec.	35	NA
180	Federal general government	35	NA
181	Federal government capital services	35	NA
182	Local government passenger transit	35	NA
183	State and local electric utilities	35	NA
184	State and local government enterprises, nec.	35	NA
185	State and local government hospitals	35	NA

186	State and local government education	35	NA
187	State and local general government, nec.	35	NA
188	State and local government capital services	35	NA
	Special Industries		
189	Noncomparable imports	NA	NA
190	Scrap, used and secondhand goods	NA	NA
191	Rest of the world industry	NA	NA
192	Inventory valuation adjustment	NA	NA

Table 2.3 35-Order Industries and Mapping to 4 Sectors

35-ORDER INDUSTRY NUMBER	4-SECTOR MAPPING	DESCRIPTION
1	D	Agriculture
2	D	Metal mining
3	D	Coal mining
4	D	Oil and gas extraction
5	D	Non-metallic mining
6	C	Construction
7	A	Food and kindred products
8	A	Tobacco
9	D	Textile mill products
10	A	Apparel
11	D	Lumber and wood
12	C	Furniture and fixtures
13	D	Paper and allied
14	D	Printing, publishing and allied
15	D	Chemicals
16	A	Petroleum and coal products
17	D	Rubber and misc plastics
18	D	Leather
19	D	Stone, clay, glass
20	D	Primary metal
21	D	Fabricated metal
22	C	Machinery, non-electrical
23	C	Electrical machinery
24	C	Motor vehicles
25	C	Transportation equipment & ordnance
26	C	Instruments
27	C	Misc. manufacturing
28	A	Transportation
29	A	Communications
30	A	Electric utilities
31	A	Gas utilities
32	A	Trade
33	A	Finance Insurance and Real Estate
34	A	Services
35	A	Government enterprises

Table 2.4 Regression Results

SIMULTANEOUS NONLINEAR LEAST SQUARES REGRESSION		
VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR
γ_c	0.8081	0.0041
γ_d	0.6182	0.0044
$\ln(\varepsilon_c)$	1.5737	0.0555
$\ln(\varepsilon_d)$	1.7881	0.0657
γ_1/γ_3	0.2010	0.0034
γ_3	0.2254	0.0230
$\ln(\varepsilon_a)$	2.0684	0.0206
θ_a	0.1464E-05	0.8336E-07
LABOR ADJUSTMENT COST REGRESSION		
VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR
α_1	0.0159	0.0053
α_2	0.0181	0.0009
COBB-DOUGLAS PREFERENCE PARAMETER REGRESSION		
VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR
μ	0.2984	0.0348

Table 2.5 Parameters: Values and Descriptions

PARAMETER	VALUE	DESCRIPTION
γ_1^a	0.0451	Sector A: CD production parameter
γ_2^a	0.7295	Sector A: CD production parameter
γ_3^a	0.2254	Sector A: CD production parameter
γ_c	0.8081	Sector C: CD production parameter
γ_d	0.6182	Sector D: CD production parameter
ε_a	7.9121	Sector A: technical change parameter
ε_c	4.8245	Sector C: technical change parameter
ε_d	5.9781	Sector D: technical change parameter
θ_a	0.1464E-05	Sector A: hourly labor in Ka production parameter
θ_b	0.1464E-05	Sector B: hourly labor in Kb production parameter
δ_{n1}	0.05	Sector A: natural attrition rate of salaried workers
δ_{ka}	0.1	Sector A: capital depreciation rate
δ_{kb}	0.1	Sector A: capital depreciation rate
α_1	0.0159	Sector A: adjustment cost parameter (net hiring squared)
α_2	0.0181	Sector A: adjustment cost parameter (gross hiring squared)
μ	0.2984	Consumer: CD preference parameter
ψ	0.25	Probability of being caught shirking
$g(\sigma)$	1.01	Extra utility from shirking

Table 2.6 Exogenous Variables: Values and Descriptions

EXOGENOUS VARIABLE	**VALUE**	**DESCRIPTION**
P	1	Numeraire price of consumer good produced in sector A
$\overline{L1}$	6	Required hours for salaried workers in sector A
Pop	91634000	Population
E_1	10	Flextime endowment for salaried workers
E_2	16	Time endowment for hourly workers ($E_2 = E_1 + \overline{L1}$)
r	0.05	Interest rate
τ_q	0.05	Sales tax on the consumer good
τ_d	0.293	Dividends tax
τ_w	0.257	Wage tax on implied wages for salaried workers
τ_{L1}	0	Flextime wage tax on salaried workers
τ_{L2}	0.257	Wage tax on hourly workers
τ_{xm}	0.05	Intermediate goods tax
τ_{xk}	0.05	Raw capital tax

Table 2.7 Endogenous Variables: Steady State Results and Descriptions

ENDOGENOUS VARIABLE	STEADY STATE VALUE	DESCRIPTION
Q_{net}	6.84E+08	Sector A: net output (consumer goods)
C	3.09E+06	Sector A: labor adjustment cost (lost output)
H_1	1.31E+04	Sector A: salaried labor hiring rate
N_1	2.62E+05	Sector A: number of salaried workers
L_1^a	1.04	Sector A: flextime hours used per salaried worker
X_m^a	2.44E+08	Sector A: intermediate goods used in production
I_a	8.10E+06	Sector A: investment
K_a	8.10E+07	Sector A: capital created and used in production
X_k^a	8.10E+06	Sector A: raw capital used in capital formation
L_2^a	9.61E+07	Sector A: hourly labor used in capital formation
D_a	3.40E+08	Sector A: gross dividends
ρ	2.13	Rental price of capital services K_b
I_b	2.71E+06	Sector B: investment
K_b	2.71E+07	Sector B: capital services produced
X_k^b	2.71E+06	Sector B: raw capital used in capital formation
L_2^b	1.07E+07	Sector B: hourly labor used in capital formation
D_b	3.75E+07	Sector B: gross dividends
X_k	1.08E+07	Sector C: raw capital produced
K_b^c	5.87E+05	Sector C: capital services used in production
L_2^c	3.08E+06	Sector C: hourly labor used in production
P_k	.632	Price of raw capital
X_m	2.44E+08	Sector D: intermediate goods produced
K_b^d	2.65E+07	Sector D: capital services used in production
L_2^d	5.34E+07	Sector D: hourly labor used in production
P_m	.634	Price of intermediate goods
Q_1	64.1	Salaried Workers: consumption
J_1	8.96	Salaried Workers: leisure
S_1	2.49	Salaried Workers: lump sum government subsidy
L_s^1	1.04	Salaried Workers: flextime labor supply
W_{L1}	17.7	Salaried Workers: flextime wage rate
W	9.78	Salaried Workers: wage rate for required hours
Y_1	226	Salaried Workers: full income
Q_2	7.30	Hourly Workers: consumption
J_2	14.2	Hourly Workers: leisure
L_s^2	1.79	Hourly Workers: labor supply
W_{L2}	1.71	Hourly Workers: wage rate
S_2	2.49	Hourly Workers: lump sum government subsidy

Y_2	25.7	Hourly Workers: full income
G	2.28E+08	Government expenditure

Appendices

APPENDIX 1.1

In order to confirm that the model is saddle path stable, first define the equations of motion as:

$$1.31 \qquad \dot{r} = \frac{(\rho+\delta)\left[-\bar{x}g'\left(N(1-r)\bar{x}\right)+\dfrac{\partial c}{\partial r}\right]+\theta\bar{x}h'(s)}{N\bar{x}^2 g''\left(N(1-r)\bar{x}\right)+\dfrac{\partial^2 c}{\partial r^2}} \equiv f$$

$$1.32 \qquad \dot{s} = N\theta r\bar{x} - \delta s \equiv k\,.$$

And define the matrix A to be:

$$1.33 \qquad A = \begin{bmatrix} \left.\partial f/\partial r\right|_{ss} & \left.\partial f/\partial s\right|_{ss} \\ \left.\partial k/\partial r\right|_{ss} & \left.\partial k/\partial s\right|_{ss} \end{bmatrix}.$$

Evaluating these derivatives results in the matrix:

$$1.34 \qquad A = \begin{bmatrix} (\rho+\delta) & \dfrac{\theta\bar{x}h''(s)}{N\bar{x}^2 g''\left(N(1-r)\bar{x}\right)+\dfrac{\partial^2 c}{\partial r^2}} \\ N\theta\bar{x} & -\delta \end{bmatrix}.$$

The stability of the system depends on the eigenvalues of the A matrix. If the eigenvalues of A are both real and have opposite signs, then the system is saddle path stable. The following equation can be used to find the eigenvalues (ε) of a matrix:

$$1.35 \qquad \varepsilon = \frac{\mathrm{Tr}(A) \mp \sqrt{\mathrm{Tr}^2(A) - 4\mathrm{Det}(A)}}{2}.$$

If the determinant of A is less than zero, then according to equation 1.35 the eigenvalues will be real and have opposite signs. The determinant of matrix A is:

$$1.36 \qquad \mathrm{Det}(A) = (\rho + \delta)(-\delta) - \frac{N\theta^2 \bar{x}^{-2} h''(s)}{N\bar{x}^{-2} g''\left(N(1-r)\bar{x}\right) + \frac{\partial^2 c}{\partial r^2}} < 0.$$

The first term is less than zero since r and δ are both positive. The numerator of the second term is positive since N is greater than zero and h''(s) is assumed to be greater than zero. The first term of the denominator is positive since g''() is also assumed to be greater than zero. Finally the second term of the denominator, $\frac{\partial^2 c}{\partial r^2}$, is assumed to be greater than zero, so the entire second term of the expression is positive. This means the determinant of A is a negative number minus a positive number and the result is less than zero. A negative determinant implies that the eigenvalues are real and have opposite signs, which in turn implies that the model is saddle path stable.

APPENDIX 1.2

The isoclines in the phase diagram represent the places where the one of the equations of motion is equal to zero. Setting the equation of motion for s equal to zero results in:

1.37 $\dot{s} = 0 \Rightarrow N\theta r\bar{x} = \delta s$.

Totally differentiating this expression yields:

1.38 $N\theta\bar{x}dr = \delta ds$.

Rearranging equation 1.38, we find the slope of the $\dot{s} = 0$ isocline:

1.39 $\frac{dr}{ds} = \frac{\delta}{N\theta\bar{x}} > 0$.

The $\dot{s} = 0$ isocline is upward sloping, linear and passes through the origin.

Now turning to the equation of motion for r and setting it equal to zero we find:

1.40 $\dot{r} = 0 \Rightarrow (\rho + \delta)\left[-\bar{x}g'\left(N(1-r)\bar{x}\right) + \frac{\partial c}{\partial r}\right] = -\theta\bar{x}h'(s)$.

Totally differentiating this expression resulting in equation 1.41:

1.41 $$(\rho+\delta)\left[N\bar{x}^2 g''\left(N(1-r)\bar{x}\right)+\frac{\partial^2 c}{\partial r^2}\right]dr = -\theta\bar{x}h''(s)ds.$$

From this equation we find that the slope of the $\dot{c}=0$ is:

1.42 $$\frac{dr}{ds}=\frac{-\theta\bar{x}h''(s)}{(\rho+\delta)\left[N\bar{x}^2 g''\left(N(1-r)\bar{x}\right)+\frac{\partial^2 c}{\partial r^2}\right]}<0.$$

The numerator of this expression is less than zero since θ, $\bar{x}$ and h''(s) are positive. In the denominator, ρ and δ are both positive, N and g''() are both greater than zero and $\frac{\partial^2 c}{\partial r^2}$ is positive. This means the denominator is positive and the entire expression is less than zero. The $\dot{r}=0$ isocline is downward sloping; however, no more can be said about its shape without assuming something about the functional forms of g(), h() and c().

APPENDIX 1.3

The signs of ds/dθ and dr/dθ indicate how the model responds to a change in θ. First, focusing on dr/dθ expressed in terms of the partial derivatives of equations 1.12 and 1.13:

1.43 $$\frac{dr}{d\theta} = \frac{-f_\theta + \dfrac{f_s k_\theta}{k_s}}{f_r - \dfrac{f_s k_r}{k_s}}.$$

Writing out those partial derivatives and signing them based on the assumptions about the variables and functions in this model we find:

1.44 $$\begin{aligned} k_r &= N\theta\bar{x} > 0 \\ k_s &= -\delta < 0 \\ k_\theta &= Nr\bar{x} > 0 \\ \\ f_r &= (\rho + \delta) > 0 \\ f_s &= \theta\bar{x}h''(s) > 0 \\ f_\theta &= \bar{x}h'(s) > 0 \end{aligned}$$

Using the signs of these partial derivatives it is clear that dr/dθ is less than zero. A decrease in θ will result in an increase in r.

Now turning our attention to ds/dθ, shown in equation 1.45 expressed in terms of the partial derivatives of equations 1.12 and 1.13:

1.45 $$\frac{ds}{d\theta} = \frac{\dfrac{k_\theta f_r}{k_r} - f_\theta}{-\dfrac{k_s f_r}{k_r} + f_s}.$$

With the signs of these partial derivatives from 1.44, it is clear that the sign of ds/dθ is indeterminate. The sign of ds/dθ depends on the sign of the numerator of 1.45. This condition is shown in equation 1.46:

1.46 $$\frac{ds}{d\theta} > 0 \quad \text{iff} \quad \frac{r}{\theta}(\rho+\delta)\left[N\bar{x}^2 g''\left(N(1-r)\bar{x}\right) + \frac{\partial^2 c}{\partial r^2}\right] - \bar{x}h'(s) > 0.$$

Expressing this as a condition on θ:

1.47 $$\frac{ds}{d\theta} > 0 \quad \text{iff} \quad \theta < \frac{r(\rho+\delta)\left[N\bar{x}^2 g''\left(N(1-r)\bar{x}\right) + \dfrac{\partial^2 c}{\partial r^2}\right]}{\bar{x}h'(s)}.$$

Unfortunately, this expression does not lend itself to a simple interpretation.

We can also learn about ds/dθ and dr/dθ from changes in the phase diagram due to a change in θ. We already found expressions for the slopes of the $\dot{s} = 0$ and $\dot{r} = 0$ isoclines in Appendix 1.2. Restating those results, the slope of the $\dot{s} = 0$ isocline is:

1.48 $$\frac{dr}{ds} = \frac{\delta}{N\theta\bar{x}} > 0 .$$

And the slope of the $\dot{r} = 0$ isocline is:

1.49 $$\frac{dr}{ds} = \frac{-\theta\bar{x}h''(s)}{(\rho + \delta)\left[N\bar{x}^{2}g''\left(N(1-r)\bar{x}\right) + \frac{\partial^{2}c}{\partial r^{2}}\right]} < 0 .$$

A decrease in θ will result in an increase in the slope of the $\dot{s} = 0$ isocline making it steeper. The slope of the $\dot{r} = 0$ isocline also increases as a result of a decrease in θ. Since the slope is negative the change makes the isocline shallower. The resulting phase diagram is shown in Figure 1.2.

APPENDIX 1.4

The model contains a third equation of motion that governs the evolution of λ, the current value of marginal damages from the stock-pollutant. From the first order condition in 1.8:

1.50 $$\dot{\lambda} = (r + \delta)\lambda + h'(s).$$

Now we rearrange the first order condition in equation 1.7 to isolate λ and substitute that into 1.50:

1.51 $$\dot{\lambda} = \frac{(r + \delta)\left[-\bar{x}g'\left(N(1-c)\bar{x}\right) + \frac{\partial j}{\partial c}\right]}{\theta\bar{x}} + h'(s).$$

This equation describes how λ evolves over time, which subsequently influences how the optimal tax changes over time.

APPENDIX 1.5

In the primary story modeled in this chapter, firms emit flow pollutants. Cleaning these flow pollutants generates a stock-pollutant. In this appendix, the story is changed so that firms generate stock-pollutants, and flow pollutants are created by the cleanup of the stock-pollutants. One example of such a situation is a firm that generates solid waste as a byproduct of production. That solid waste can either be dumped in a landfill contributing to a stock-pollutant, or it can be incinerated in a way that creates a flow pollutant. With a few minor changes, the model presented in this chapter can capture this alternate story. These changes are highlighted in this appendix.

First of all, the definitions of some of our variables need to be changed to fit the new story. We still have N firms each of which emits a constant amount of pollution $\bar{x}$. However, $\bar{x}$ is now a stock-pollutant instead of a flow-pollutant. The one control variable in the model is still r. The definition of r is now the percentage of the new stock-pollutant that is not incinerated, instead of the percentage of pollution removed from the emissions. With this definitional change, the total amount of flow pollution is still $N(1-r)\bar{x}$. The equation of motion for the stock-pollutant remains the same as was stated in 1.1, with θ equal to one since all of the pollution not incinerated ends up in the stock. The g() and h() functions for damages from flow and stock-pollutants remain the same. The $c(r, \bar{x})$ function is now the cost of incineration where $\frac{\partial c}{\partial r} < 0$ and $\frac{\partial^2 c}{\partial r^2} = 0$. The sign of the first derivative is now negative instead of positive. An increase in r still implies an increase in the amount of stock-pollutant. However, in this new version of the model increasing r means a reduction in firms' pollution control activity, incineration, and a reduction in costs. Note that the sign of the second derivative has also changed from the original model. While removing a percentage of pollution from a flow most likely is subject to decreasing returns to scale, the cost of incineration is more likely to be subject to constant returns to scale.

Even with these changes, much of the model remains unaltered from the original version. The social planner's problem is identical to the original version. The Hamiltonian is unchanged along with the subsequent first order conditions and the resulting equations of motion. This means 1.2 through 1.11 remain the same. The phase diagram from Figure 1.1 is also identical in both versions of the model. The stability analysis from Appendix 1.1 and the derivation of the slopes of the isoclines found in Appendix 1.2 are unaltered as well.

Since θ is fixed at one in this version of the model, the experiment reducing θ is not relevant here. Therefore, 1.12 through 1.19 and Appendix 1.3 are not applicable to this version of the model.

The first major difference in the results of the two models is the free market outcome. In the original story, firms acting in the free market set r equal to zero and thus do not clean emissions at all. This means that the stock of pollution never forms, only the flow pollution emitted by firms exists. In the new version, firms set r equal to one in the free market and do not incinerate any of their waste. This is a similar result in that the firm takes no action to reduce its emitted pollution. However, we now have a stock pollution problem instead of a flow pollution problem under free market conditions.

The primary differences between the two versions of the model arise in the policy area. While the condition that needs to be satisfied by an optimal policy remains the same as presented in 1.20 (rewritten here as 1.52), the interpretation of this condition has changed.

1.52 $$\bar{x}g'\left(N(1-r)\bar{x}\right)+\lambda\theta\bar{x}=\frac{\partial c}{\partial r}.$$

The marginal damage from the flow pollutant for a small change in r, the percentage of pollution not incinerated, is shown in the first term on the left hand side of 1.52. The value of this term is positive. The second term is the present value of the marginal damages from the stock-pollutant for a small change in r. This term is negative since, $\theta = 1$, $\bar{x} \geq 0$ and $\lambda<0$. The term on the right hand side of this expression is the marginal cost of increasing the percentage of pollution that is not incinerated. This is a negative number since reducing incineration reduces costs. If the left hand side of 1.52 is less than zero, then an optimal level incineration, $0 < (1-r) \leq 1$, exists. If the left hand side of 1.52 is greater than or equal to zero, then this condition will not hold since $\frac{\partial c}{\partial r} < 0$. If this is the case, then the optimal policy is no incineration (i.e. set r=1). If increasing the flow pollutant causes greater harm than the benefit from reducing the stock-pollutant, then it is better to do nothing.

Now we look at the firm's problem in order to find how to set the optimal tax. Once again the firm chooses r to maximize profits after taxes and incineration costs:

$$1.53 \qquad \max_{r} f(\bar{x}) - c(r, \bar{x}) - \theta r \bar{x} \tau .$$

The tax is now on the firm's contribution to the stock-pollutant instead of the flow pollutant as in the original version. From this optimization the firm will choose r to satisfy:

$$1.54 \qquad \frac{\partial c}{\partial r} = -\theta \bar{x} \tau .$$

Using 1.52 and 1.54 we find that the new optimal tax on a firm's contribution to the stock-pollutant is:

$$1.55 \qquad \tau^{*} = -\lambda + \frac{1}{\theta} g'\left(N(1-r)\bar{x}\right).$$

Now we look at what the tax would be if the regulator ignored the flow pollutant. The optimality condition that would be used is:

$$1.56 \qquad \theta \bar{x} \lambda = \frac{\partial c}{\partial r} .$$

The tax that satisfies this condition is:

1.57 $\tau^{s} = -\lambda$.

This tax is higher than the optimal tax and the resulting level of incineration is strictly greater than the optimal level of incineration. Given the nature of the flow pollutant in this example, it is rather unlikely that a regulator actually would ignore the flow pollutant.

Transboundary issues could be very important when looking at the decision between incineration and contributing to a landfill. As in the original model, we make γ be the proportion of the flow pollutant damage that affects the upstream state. The tax set by a regulator from the upstream state would thus be:

1.58 $\tau^{up} = -\lambda + \frac{\gamma}{\theta} g'\left(N(1-r)\bar{x}\right)$.

This tax is greater than the optimal tax so the corresponding level of incineration will be higher than the optimal level. This, of course, means that the upstream state is using the tax to encourage firms to incinerate more and thus export pollution to the downstream state.

APPENDIX 2.1

In this appendix we examine the stability of sectors A and B in the theoretical model. Since there is no interaction between the capital and labor adjustment costs in sector A, we can look at the stability of the two systems separately. We begin by defining the equation of motion for hiring (2.37) to be F_h and the equation of motion for the stock of workers to be G_n (2.30). We can now define the matrix A_{labor} as:

$$2.102 \quad A_{labor} = \begin{bmatrix} \left.{}^{\partial F_h}\!\big/\!{}_{\partial H_1}\right|_{ss} & \left.{}^{\partial F_h}\!\big/\!{}_{\partial N_1}\right|_{ss} \\ \left.{}^{\partial G_n}\!\big/\!{}_{\partial H_1}\right|_{ss} & \left.{}^{\partial G_n}\!\big/\!{}_{\partial N_1}\right|_{ss} \end{bmatrix}.$$

Substituting in the above derivatives evaluated at the steady state, we find:

$$2.103 \quad A_{labor} = \begin{bmatrix} (r+\delta_{n1}) & \dfrac{-(r+\delta_{n1})\alpha_1\delta_{n1}}{\alpha_1+\alpha_2} \\ 1 & -\delta_{n1} \end{bmatrix}.$$

The stability of the system depends on the eigenvalues of the A matrix. If the eigenvalues of A are both real and have opposite signs, then the system is saddle path stable. The following equation can be used to find the eigenvalues of a matrix:

2.104 $$\lambda = \frac{Tr(A) \mp \sqrt{Tr^2(A) - 4Det(A)}}{2}.$$

If the determinant of A is less than zero, then according to equation 2.104 the eigenvalues will be real and have opposite signs. The determinant and trace of matrix A_{labor} are:

2.105 $$Tr(A_{labor}) = r > 0$$

2.106 $$Det(A_{labor}) = -\delta_{n1}(r + \delta_{n1})\frac{\alpha_2}{\alpha_1 + \alpha_2} < 0.$$

We find that the trace of A_{labor} is greater than zero, and the determinant of A_{labor} is less than zero. This implies that the eigenvalues are real and have opposite signs, which in turn implies that the model is saddle path stable.

We can also examine the stability of the capital system. Following an analogous procedure for capital, we define the equation of motion for investment (2.35) to be F_{ia} and the equation of motion for the capital stock to be G_{ka} (2.33). We can now define the matrix A_{capA} as:

2.107 $$A_{capA} = \begin{bmatrix} \left.{}^{\partial F_{ia}}\!/\!{}_{\partial I_a}\right|_{ss} & \left.{}^{\partial F_{ia}}\!/\!{}_{\partial K_a}\right|_{ss} \\ \left.{}^{\partial G_{ka}}\!/\!{}_{\partial I_a}\right|_{ss} & \left.{}^{\partial G_{ka}}\!/\!{}_{\partial K_a}\right|_{ss} \end{bmatrix}.$$

Substituting in the values for the partial derivatives results in:

$$2.108 \quad A_{capA} = \begin{bmatrix} (r+\delta_{ka}) & 0 \\ 1 & -\delta_{ka} \end{bmatrix}.$$

Looking at the trace and determinant of the matrix, we find:

$$2.109 \quad Tr(A_{capA}) = r > 0$$

$$2.110 \quad Det(A_{capA}) = -\delta_{ka}(r+\delta_{ka}) < 0\,.$$

The positive trace and negative determinant implies that the capital system is also saddle path stable.

Finally, we can look at the stability of sector B. As with the previous two analyses, we start by defining the equation of motion for investment (2.50) to be F_{ib} and the equation of motion for the capital stock to be G_{kb} (2.49). As before, we now define the matrix A_{capB} as:

$$2.111 \quad A_{capB} = \begin{bmatrix} \left.\partial F_{ib}\middle/\partial I_b\right|_{ss} & \left.\partial F_{ib}\middle/\partial K_b\right|_{ss} \\ \left.\partial G_{kb}\middle/\partial I_b\right|_{ss} & \left.\partial G_{kb}\middle/\partial K_b\right|_{ss} \end{bmatrix}.$$

Substituting in the values for the partial derivatives results in:

$$2.112 \quad A_{capB} = \begin{bmatrix} (r+\delta_{kb}) & 0 \\ 1 & -\delta_{kb} \end{bmatrix}.$$

Looking at the trace and determinant of the matrix, we find:

2.113 $\quad Tr(A_{capB}) = r > 0$

2.114 $\quad Det(A_{capB}) = -\delta_{kb}(r + \delta_{kb}) < 0$.

The positive trace and negative determinant implies that the capital system is also saddle path stable.

APPENDIX 2.2

By setting the equations of motion for hiring (2.37) and the salaried labor stock (2.30) equal to zero and rearranging them, we can find equations describing the $\dot{H}_1 = 0$ and $\dot{N}_1 = 0$ isoclines. The following equation tells us about the $\dot{N}_1 = 0$ isocline:

$$2.115 \quad \dot{N}_1 = 0 \Rightarrow H_1 = \delta_{n1} N_1 .$$

This equation tells us that the $\dot{N}_1 = 0$ isocline is linear, passes through the origin, and has a slope equal to δ_{n1}. The equation below describes the $\dot{H}_1 = 0$ isocline:

$$2.116 \quad \dot{H}_1 = 0 \Rightarrow H_1 = \frac{\frac{\overline{L}_1}{P}(W_{L1} - W)}{2(\alpha_1 + \alpha_2)} + \frac{\alpha_1 \delta_{n1}}{(\alpha_1 + \alpha_2)} N_1 \ .$$

We can see that the $\dot{H}_1 = 0$ isocline is also linear. The second term on the right hand side of the equation is that the slope is positive and less than the slope of the $\dot{N}_1 = 0$ isocline. The first term on the right hand side tells us where the isocline intercepts the H_1 axis. This term is positive if the difference between the flextime wage and the base wage is positive. Note that this is the same condition that tells us that H_1 and N_1 are positive in the steady state.

With this information about the $\dot{H}_1 = 0$ and $\dot{N}_1 = 0$ isoclines, we can place the isoclines in Figure 2.1. The next step in constructing the phase diagram is to see what happens away from the isoclines. Looking at the equation of motion for

N_1 (2.30), we can see that if we step away from the isocline by increasing H_1, then N_1 will rise. If we decrease H_1, then N_1 will fall. This gives us the horizontal arrows in the four quadrants of the phase diagram. Turning to the H_1 equation of motion (2.34), we see that if we step off of the isocline by increasing H_1, then H_1 begins to rise, and the opposite happens if we look below the H_1 isocline. We can now complete the directional arrows in the four quadrants of the phase diagram. The final step is to verify that the model is saddle path stable, which is done in Appendix 2.1, and draw in the saddle path, which completes the phase diagram.

The I-K phase diagrams for sectors A and B have similar derivations. By setting the equations of motion for capital in sectors A (2.33) and B (2.49) equal to zero and rearranging, we can find the following equations that describe their respective isoclines:

2.117 $\dot{K}_a = 0 \Rightarrow I_a = \delta_{ka} K_a$

2.118 $\dot{K}_b = 0 \Rightarrow I_b = \delta_{kb} K_b$.

These equations tell us that, for sectors A and B, the $\dot{K}_a = 0$ and $\dot{K}_b = 0$ isoclines are linear, pass through their respective origins, and both have slopes equal to the decay rate of capital. Turning to the $\dot{I}_a = 0$ and $\dot{I}_b = 0$ isoclines, we set the equations of motion for investment in the two sectors (2.35 and 2.50) equal to zero and rearranging we find:

$$2.119 \quad \dot{I}_a = 0 \Rightarrow I_a = \frac{1}{2W_{L2}\theta_a}\left[\frac{\gamma_2 (\varepsilon_a P)^{\frac{1}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}}{(r+\delta_{ka})} - P_k\right]$$

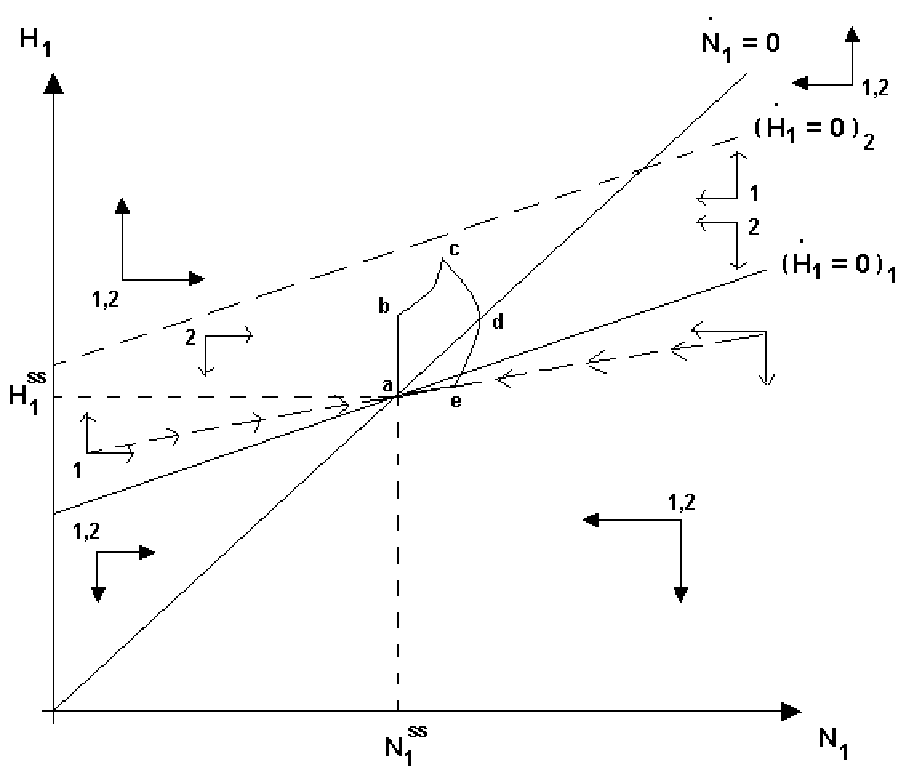
$$2.120 \quad \dot{I}_b = 0 \Rightarrow I_b = \frac{1}{2W_{L2}\theta_b}\left[\frac{\rho}{(r+\delta_{kb})} - P_k\right].$$

These equations tell us that both the $\dot{I}_a = 0$ and $\dot{I}_b = 0$ isoclines are linear and have slopes of zero. The only difference between the two of them is where they intercept the I_a and I_b axes. We can now place the isoclines in their respective phase diagrams. We can find the directional arrows to place in the four quadrants of the phase diagrams by following the same procedure used for the H_1-N_1 phase diagram. These systems were shown to be saddle path stable in Appendix 2.1, so we can place the saddle paths to complete the phase diagrams.

APPENDIX 2.3

We can use the phase diagrams to see how the firm responds to a variety of exogenous shocks. Here we will show the results of an announced *temporary* decrease in τ_w for salaried workers. The phase diagram and integral curves for this experiment are shown in Figure 2.8:

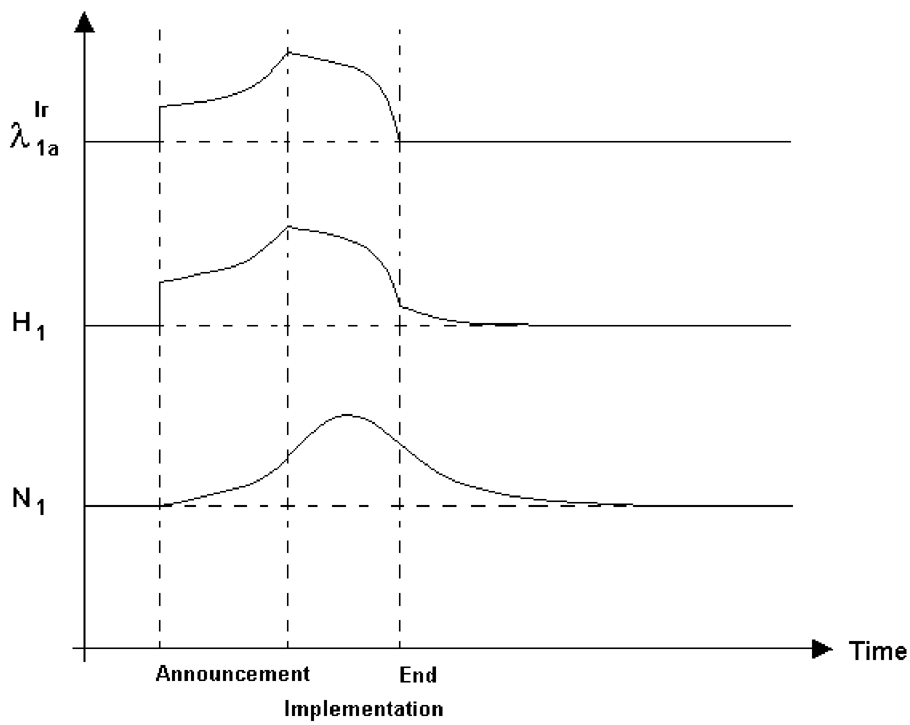
Figure 2.8 Effects of an Announced Temporary Decrease of τ_w



In the phase diagram above, the firm begins at the steady state equilibrium shown by point a. As soon as the announcement is made that a temporary decrease in the tax rate on the implied hourly wage rate for salaried workers, the firm increases its hiring rate and jumps to point b. In the period between the announcement of the policy and the implementation of the policy, the firms hiring rate and stock of employees evolves along a path dictated by the original isoclines. This path is shown as the curve between points b and c where both the hiring rate and the stock of employees are increasing. When the new policy is actually implemented and the τ_w falls, the $\dot{H}_1 = 0$ isocline shifts up from $(\dot{H}_1 = 0)_1$ to $(\dot{H}_1 = 0)_2$, and the evolution of the hiring rate and the capital stock is governed by the new isoclines. While the policy is in place, the hiring rate and the stock of employees evolves along the path between the points c and e. Initially, the hiring rate begins to decline while the stock of employees continues to rise. At point d, the path crosses the $\dot{N}_1 = 0$ isocline and the stock of employees begins to fall. When the policy ends, the model is again governed by the original isoclines and the hiring rate and stock of employees evolve along the saddle path from point e back to point a.

The integral curves for the evolution of the state variable N_1, the control variable H_1, and the costate variable λ_{1a}^{lr} are shown in Figure 2.9. The integral curves tell something about the speed of the adjustments shown in the phase diagram. In Figure 2.9, the announcement corresponds to the jump from point a to point b in Figure 2.8. The implementation of the policy corresponds to point c in the phase diagram. Point d of the phase diagram corresponds to the peak value of N_1, which occurs between the implementation of the policy and the end of the policy. Point e corresponds with the end of the policy. Finally, the model approaches to point a as the time becomes very large.

Figure 2.9 Integral Curves for Announced Temporary Decrease of τ_w



In this sort of experiment, we see the importance of the intertemporal perspective. In the steady state, nothing has changed; however, the dynamic path shows that the firm is undergoing many changes in the intermediate periods.

APPENDIX 2.4

This appendix lists all of the equations included in the steady state computational model. The equations included from sector A are: the factor demand for intermediate goods:

$$2.12 \qquad X_m^a = (K_a)(\varepsilon_a P)^{\frac{1}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}} \right)^{\frac{\gamma_1^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m} \right)^{\frac{1-\gamma_1^a}{\gamma_2^a}},$$

the factor demand for flextime labor:

$$2.13 \qquad N_1 L_1^a = (K_a)(\varepsilon_a P)^{\frac{1}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}} \right)^{\frac{1-\gamma_3^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m} \right)^{\frac{\gamma_3^a}{\gamma_2^a}} - \bar{L}_1 N_1,$$

the condition imposing price equal to marginal cost:

$$2.11 \qquad Q_{net} = \varepsilon_a K_a (\varepsilon_a P)^{\frac{1-\gamma_2^a}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}} \right)^{\frac{\gamma_1^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m} \right)^{\frac{\gamma_3^a}{\gamma_2^a}} - C(H_1, N_1),$$

the functional form of adjustment costs:

$$2.36 \qquad C(H_1, N_1) = \alpha_1 (H_1 - \delta_{n1} N_1)^2 + \alpha_2 (H_1)^2,$$

the investment and raw capital relationship:

2.16 $X_k^a = I_a$,

the investment and hourly labor relationship:

2.17 $L_2^a = \theta_a (I_a)^2$,

a new equation describing gross dividends (D_a):

2.121 $D_a = PQ_{net} - W\overline{L}_1 N_1 - W_{L1} N_1 L_1^a - P_m X_m^a - P_k X_k^a - W_{L2} L_2^a$,

and the four equations of motion for H_1, N_1, I_a, and K_a, all set equal to zero:

2.37 $$\dot{H}_1 = \frac{(r+\delta_{n1})(2\alpha_1(H_1 - \delta_{n1}N_1) + 2\alpha_2 H_1) + \frac{1}{P}\left[W\overline{L}_1 - W_{L1}\overline{L}_1\right]}{2(\alpha_1 + \alpha_2)} = 0$$

2.30 $$\dot{N}_1 = H_1 - \delta_{n1} N_1 = 0$$

2.35 $$\dot{I}_a = \frac{(r+\delta_k)[P_k + 2W_{L2}\theta_a I_a] - \gamma_2 (\varepsilon_a P)^{\frac{1}{\gamma_2^a}} \left(\frac{\gamma_1^a}{W_{L1}}\right)^{\frac{\gamma_1^a}{\gamma_2^a}} \left(\frac{\gamma_3^a}{P_m}\right)^{\frac{\gamma_3^a}{\gamma_2^a}}}{2W_{L2}\theta_a} = 0$$

2.33 $$\dot{K}_a = I_a - \delta_{ka} K_a = 0.$$

From sector B the steady state computational model includes the following equations: the raw capital factor demand equation:

2.44 $X_k^b = I_b$,

the hourly labor factor demand:

2.45 $L_2^b = \theta_b (I_b)^2$,

the equations of motion for I_b and K_b set equal to zero:

2.50 $$\dot{I}_b = \frac{(r + \delta_{kb})(P_k + W_{L2}\theta_b I_b) - \rho}{2W_{L2}\theta_b} = 0$$

2.49 $$\dot{K}_b = I_b - \delta_{kb} K_b = 0,$$

and a new expression for gross dividends (D_b):

2.122 $D_b = \rho K_b - P_k X_k^b - W_{L2} L_2^b$.

The steady state computational model also includes the following equations from sectors C and D: their factor demands for capital services:

2.54 $$K_b^c = \frac{1}{\varepsilon_c} X_k \left(\frac{W_{L2}(1-\gamma_c)}{\rho\gamma_c} \right)^{\gamma_c}$$

2.58 $$K_b^d = \frac{1}{\varepsilon_d} X_m \left(\frac{W_{L2}(1-\gamma_d)}{\rho\gamma_d} \right)^{\gamma_d},$$

their factor demands for hourly labor:

2.55 $$L_2^c = \frac{1}{\varepsilon_c} X_k \left(\frac{\rho \gamma_c}{W_{L2}(1-\gamma_c)} \right)^{1-\gamma_c}$$

2.59 $$L_2^d = \frac{1}{\varepsilon_d} X_m \left(\frac{\rho \gamma_d}{W_{L2}(1-\gamma_d)} \right)^{1-\gamma_d},$$

and the zero profit conditions:

2.56 $$P_k X_k = (W_{L2} L_2^c + \rho K_b^c)(1+\tau_{xk})$$

2.60 $$P_m X_m = (W_{L2} L_2^d + \rho K_b^d)(1+\tau_{xm}).$$

For the two consumer types, the steady state computational model includes: their demand equations for the consumption good:

2.63 $$P(1-\tau_q)Q_1 = \mu Y_1$$

2.68 $$P(1-\tau_q)Q_2 = \mu Y_2,$$

their demand equations for leisure:

2.64 $$W_{L1}(1-\tau_{L1})J_1 = (1-\mu)Y_1$$

2.69 $$W_{L2}(1-\tau_{L2})J_2 = (1-\mu)Y_2,$$

their labor supply equations:

2.65 $$W_{L1}(1-\tau_{L1})L_1^s = \mu Y_1 - W(1-\tau_w)\overline{L}_1 - S_1 - \frac{(D_a + D_b)(1-\tau_d)}{Pop}$$

2.70 $$W_{L2}(1-\tau_{L2})L_2^s = \mu Y_2 - S_2 - \frac{(D_a + D_b)(1-\tau_d)}{Pop},$$

and finally, their full income definitions:

2.62 $$Y_1 \equiv W(1-\tau_w)\overline{L}_1 + W_{L1}(1-\tau_{L1})E_1 + S_1 + \frac{(D_a + D_b)(1-\tau_d)}{Pop}$$

2.67 $$Y_2 \equiv W_{L2}(1-\tau_{L2})E_2 + S_1 + \frac{(D_a + D_b)(1-\tau_d)}{Pop}.$$

Three equations are included from the government sector: the government revenue equation:

2.71 $$\begin{aligned} G = {} & \tau_w W\overline{L}_1 N_1 + \tau_{L1} W_{L1} L_1^s N_1 + \tau_{L2} W_{L2} L_2^s N_2 \\ & + \tau_d(D_a + D_b) + \tau_q P(Q_1 N_1 + Q_2 N_2) + \tau_{xk} P_k X_k + \tau_{xm} P_m X_m, \end{aligned}$$

the government expenditure equation:

2.72 $$G = S_1 N_1 + S_2 N_2,$$

and the equal subsidy definition:

2.73 $$S_1 = S_2.$$

The efficiency wage condition needed to ensure salaried workers rationally choose the high level of effort:

$$2.75 \qquad \frac{(Q_1)^{\mu}(J_1)^{1-\mu}}{r} = \frac{\ln(\psi)g(\sigma)(Q_1)^{\mu}(J_1)^{1-\mu}}{-r\left(r-\ln(\psi)\right)} + \frac{(Q_2)^{\mu}(J_2)^{1-\mu}}{r-\ln(\psi)}.$$

Finally, the steady state computational model includes five of the six market clearing conditions: market-clearing conditions for both types of labor:

$$2.77 \qquad L_1^s = L_1$$

$$2.78 \qquad L_2^s N_2 = L_2^a + L_2^b + L_2^c + L_2^d,$$

the intermediate goods market clearing condition:

$$2.79 \qquad X_m = X_m^a,$$

the raw capital market clearing condition:

$$2.80 \qquad X_k = X_k^a + X_k^b,$$

and finally the capital services market clearing condition:

$$2.81 \qquad K_b = K_b^c + K_b^d\ .$$

The consumption good market clearing condition (2.76) is dropped in order to satisfy Walras' law.

APPENDIX 2.5

As a further comparison of the high and low adjustment cost cases of the temporary increase of τ_s simulation, we can look at the equivalent variations for the two cases.[41]

Table 2.8 Equivalent Variation: High and Low Labor Adjustment Costs

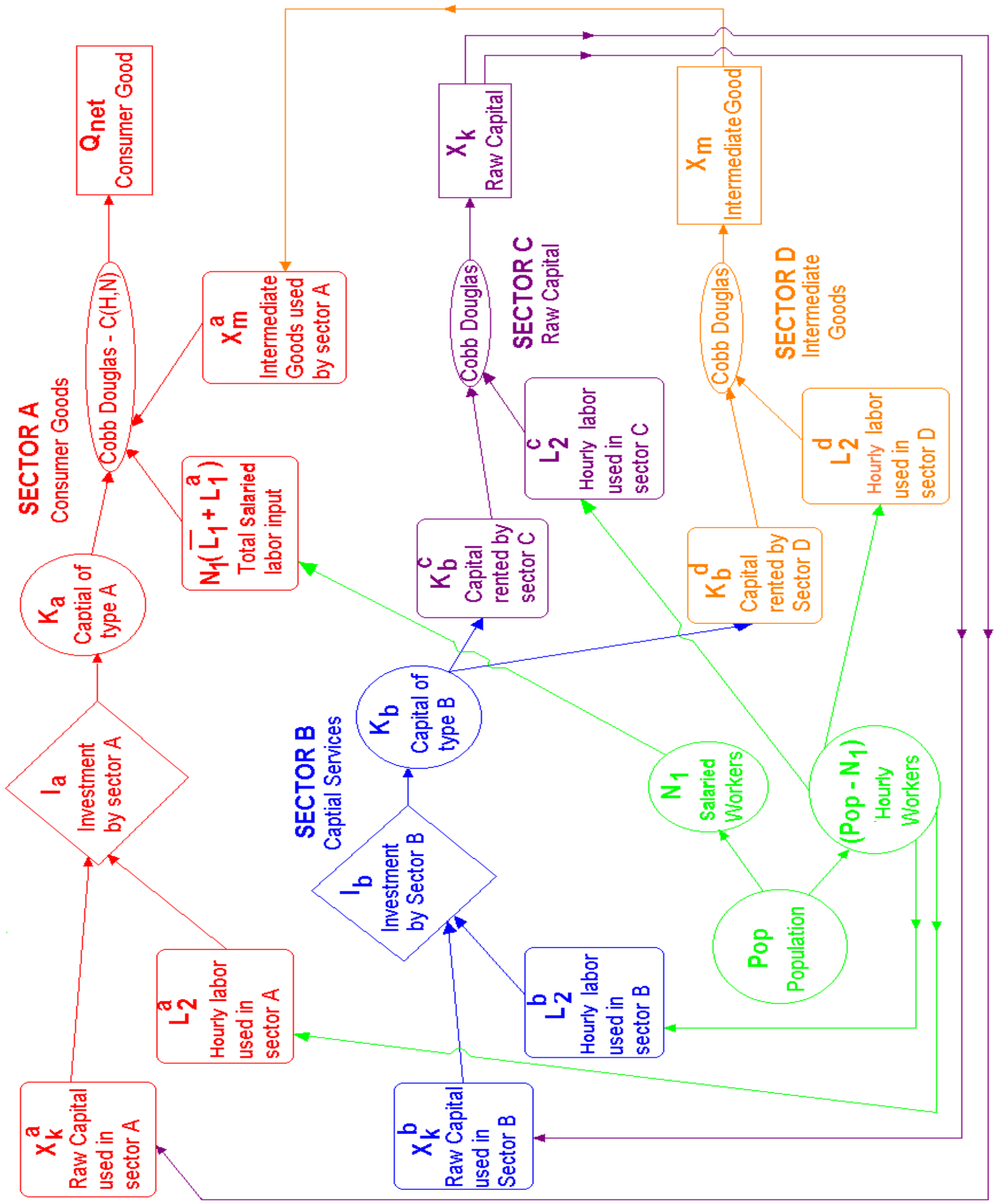
	EV	G Old	G New	G Change	MCPF
High Costs	-1.36E+07	1.86E+09	2.30E+09	4.44E+08	1.031
Low Costs	-1.49E+07	1.98E+09	2.45E+09	4.69E+08	1.032

Here we see a somewhat surprising result, the equivalent variation his larger for the case with low labor adjustment costs than it is for the case with high labor adjustment costs. In light of what actually happens to salaried workers in the two cases, this result should actually be expected. The efficiency wage condition tells us that the salaried workers will always be better off than hourly workers, and in the low labor adjustment cost case, more salaried workers are loosing their jobs and becoming hourly workers than in the high labor adjustment cost case. Clearly then, more people are harmed by the policy in the low labor adjustment cost case than in the high labor adjustment cost case, and this is born out by the equivalent

[41] It should be noted that the comparison of equivalent variations here is unlike comparing two policies. This is because the two simulations start at different base cases due to the different values assigned to the labor adjustment cost parameters.

variation results. Conversely, if the economy experiences a positive shock, then more people will benefit in the low labor adjustment cost case than in the high labor adjustment cost case. The inclusion of labor adjustment costs can have a significant impact on the analysis of a policy's equivalent variation.

APPENDIX 2.6



SECTOR A
Consumer Goods
Qnet
Consumer Good
Cobb Douglas - C(H,N)
Ka
Captial of type A
Ia
Investment by sector A
Xk a
Raw Capital used in sector A
L2 a
Hourly labor used in sector A
N1(L1 + L1 a)
Total Salaried labor input
Xm a
Intermediate Goods used by sector A
SECTOR B
Captial Services
Kb
Capital of type B
Ib
Investment by Sector B
Xk b
Raw Capital used in Sector B
L2 b
Hourly labor used in sector B
SECTOR C
Raw Capital
Xk
Raw Capital
Cobb Douglas
Kb c
Capital rented by sector C
L2 c
Hourly labor used in sector C
SECTOR D
Intermediate Goods
Xm
Intermediate Good
Cobb Douglas
Kb d
Capital rented by Sector D
L2 d
Hourly labor used in sector D
N1
Salaried Workers
(Pop - N1)
Hourly Workers
Pop
Population

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Vita

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